Analogue of many-body Berry–Esseen theorem for critical systems

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More details at

arXiv:2011.10513 **[quant-ph]**

Many-body system

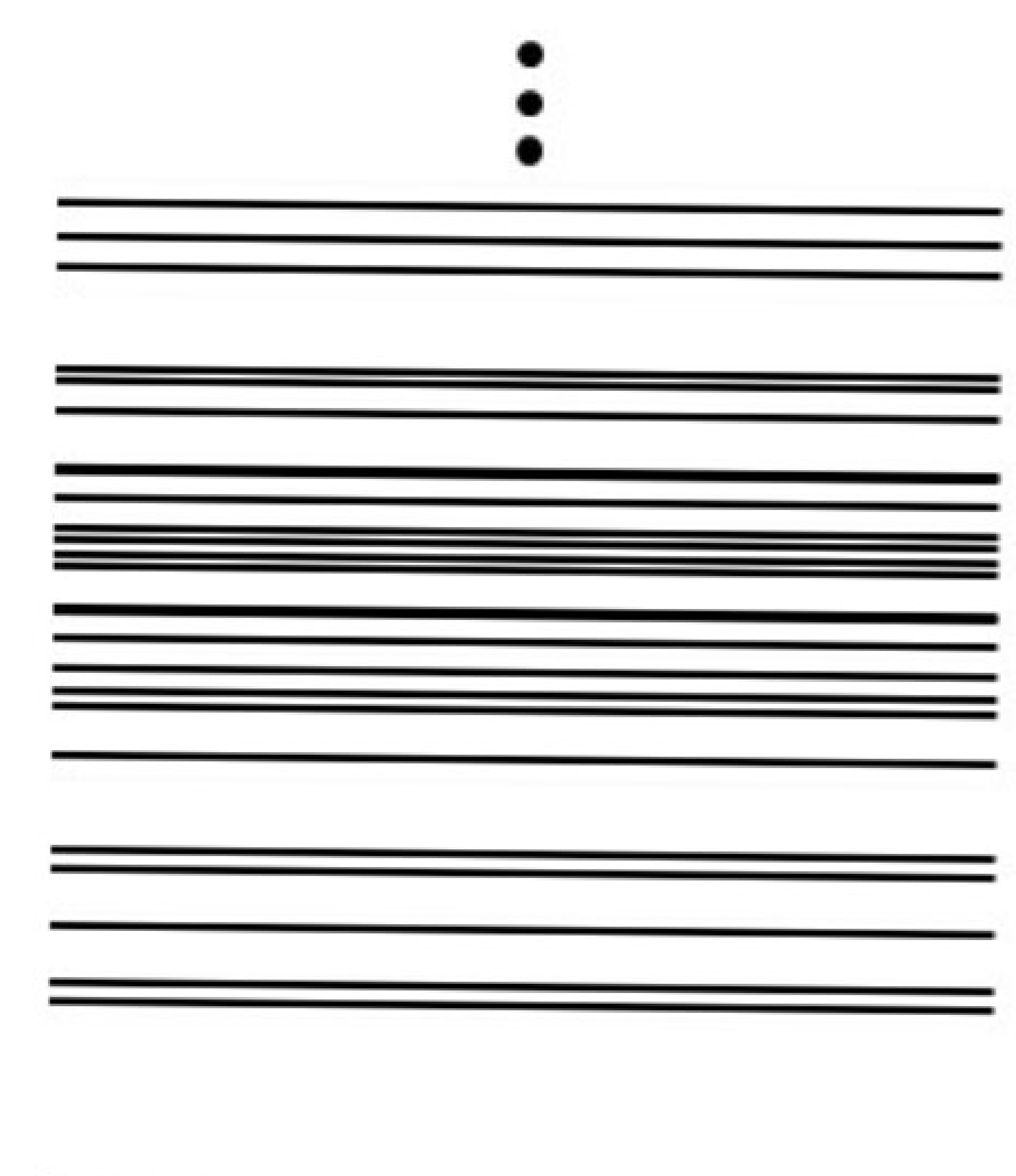
$$H = \sum_{\substack{\langle i,k\rangle \\ a,b}} [atom_i^a] \otimes [atom_k^b] + \sum_{\substack{i \\ a}} V(atom_i^a)$$

Complicated Hamiltonian

$$H = \sum_{\substack{\langle i,k \rangle \\ a,b}} [atom_i^a] \otimes [atom_k^b] + \sum_{\substack{i \\ a}} V(atom_i^a)$$

$\downarrow$

Complicated Spectrum

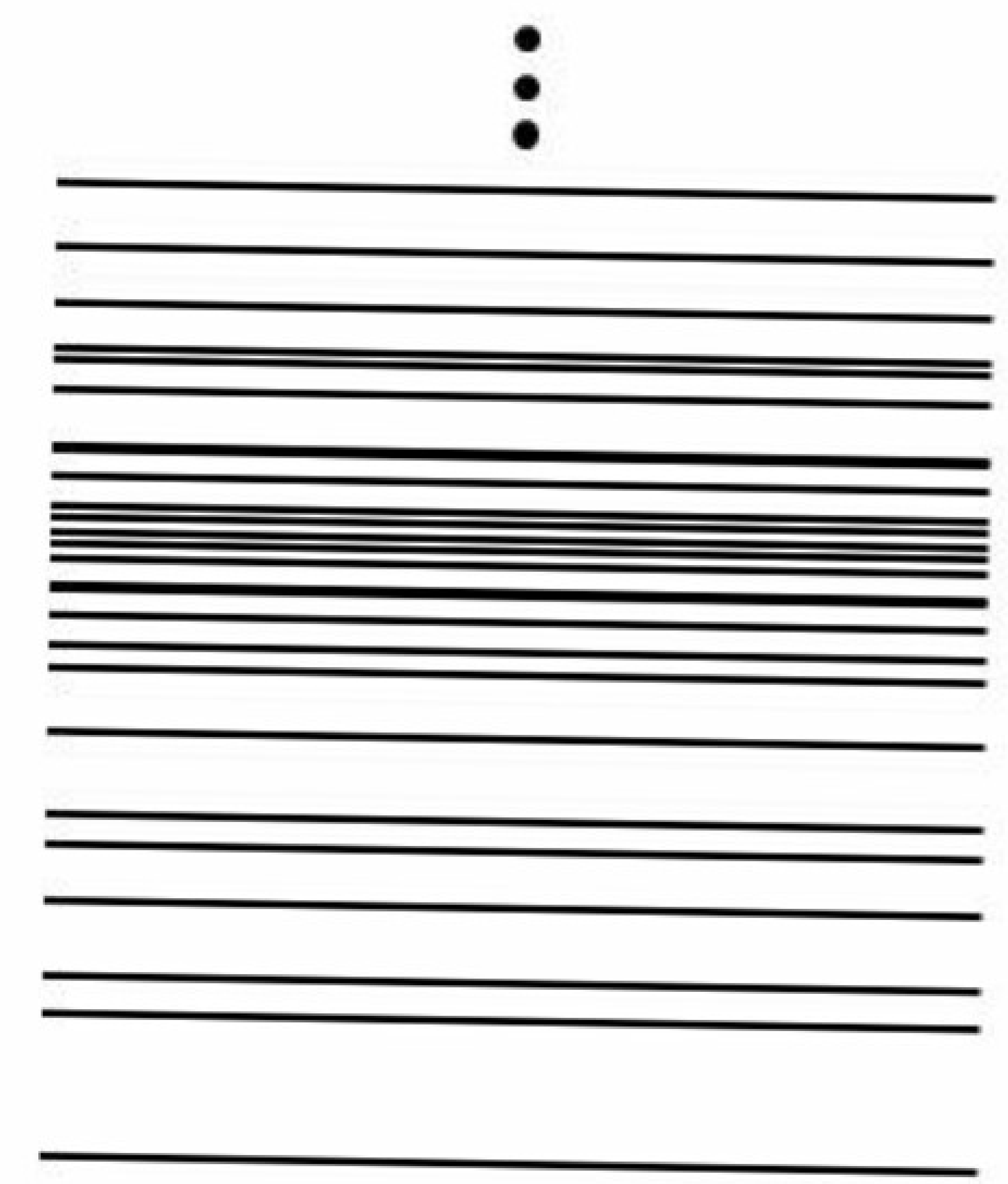


Simple Hamiltonian

$\Big\downarrow$

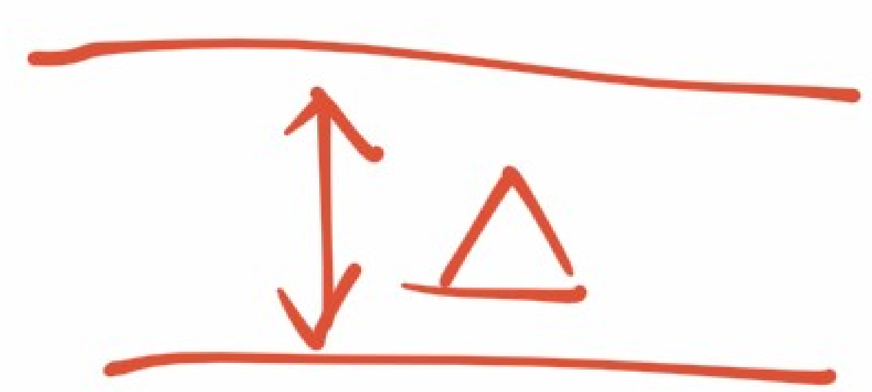
Complicated Spectrum

$$H = J \sum_{\langle i,i+1 \rangle} \sigma_x^i \otimes \sigma_x^{i+1} + h \sum_i \sigma_z^i$$



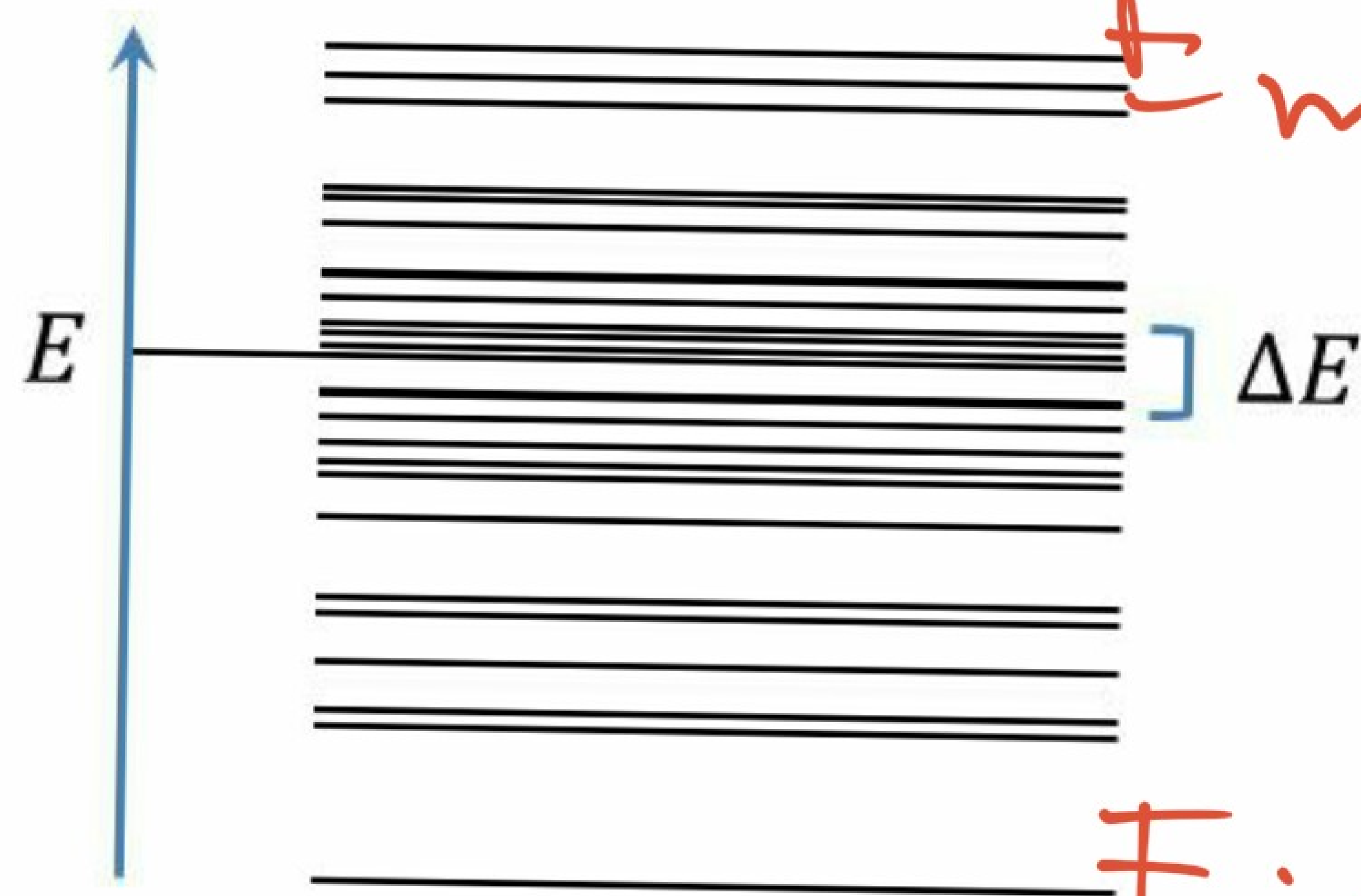
Even the question about the spectral gap is hard; in fact, undecidable.

Cubitt, Perez-Garcia, & Wolf, Nature 528, 207 (2015)
Bausch, Cubitt, Lucia, & Perez-Garcia, PRX 10, 031038 (2020)



No

Density is easier



E_m

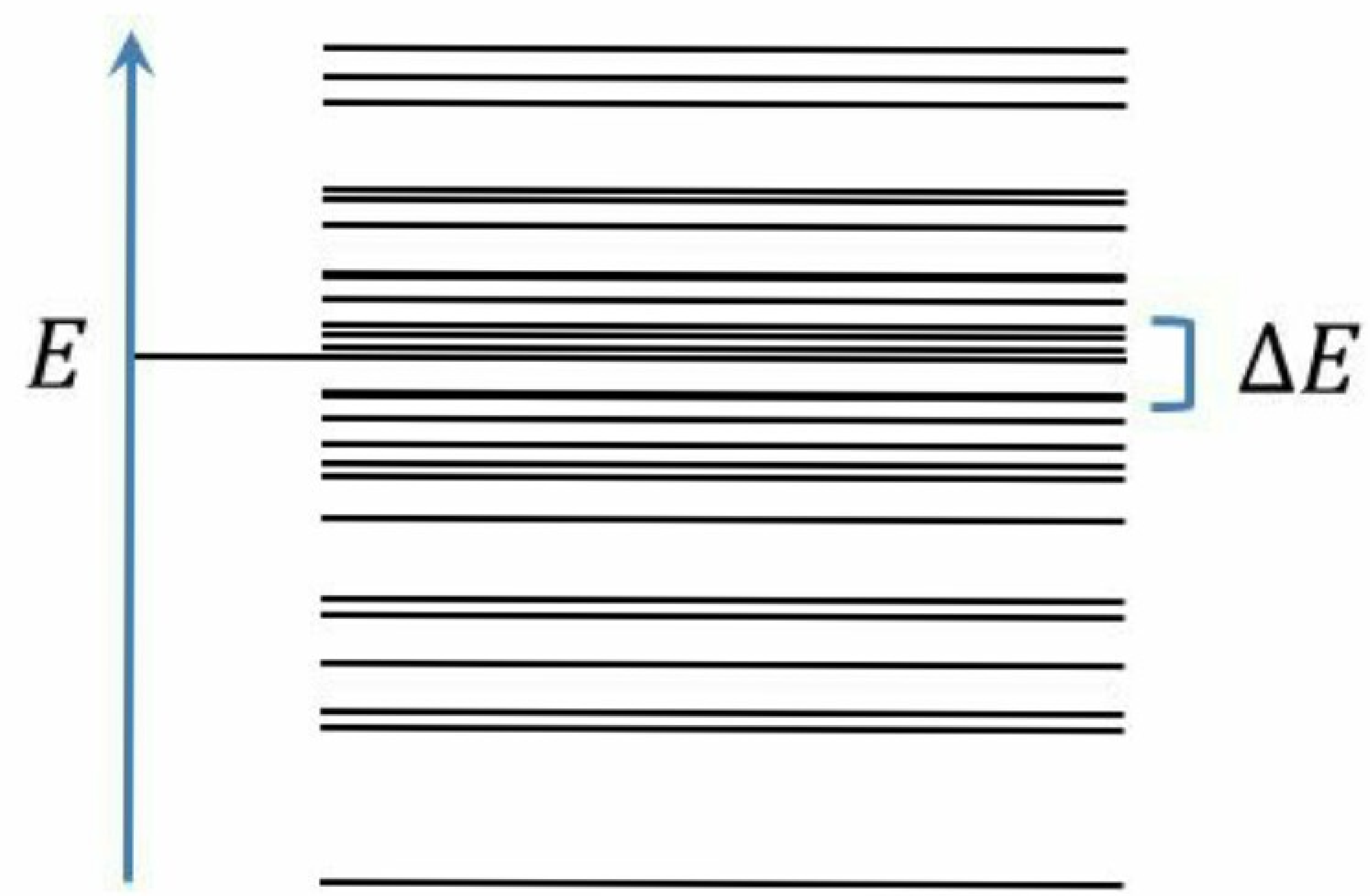
E_{min}

$$\Delta N = \Omega(E)\Delta \mathrm{E}$$

$$H = \sum_{ij} A_i \otimes A_j$$

N $\quad d^N$, $\quad E_m \sim N$

Density is easier



$$\Delta N = \Omega(E)\Delta E$$

$$\boxed{\tau = e^{-\beta H}/Z}$$

$$p_i = \frac{e^{-\beta E_i}}{Z}$$

$$\rho(E)dE = \Omega(E)\frac{e^{-\beta E}}{Z}dE$$

Many-body Berry-Esseen

$p(E) = \Omega(E) \frac{e^{-\beta H}}{Z}$

$F(E) = \int_{E_{min}}^{E} dE' \, p(E')$

$g(E) = \frac{e^{-\frac{(H - \langle H \rangle)^2}{\sigma(H)}}}{Norm}$

$$\frac{\langle X \otimes Y \rangle - \langle X \rangle \langle Y \rangle}{\|X\| \|Y\|} = e^{-l/\xi}$$

$$\sigma(H) = sN$$

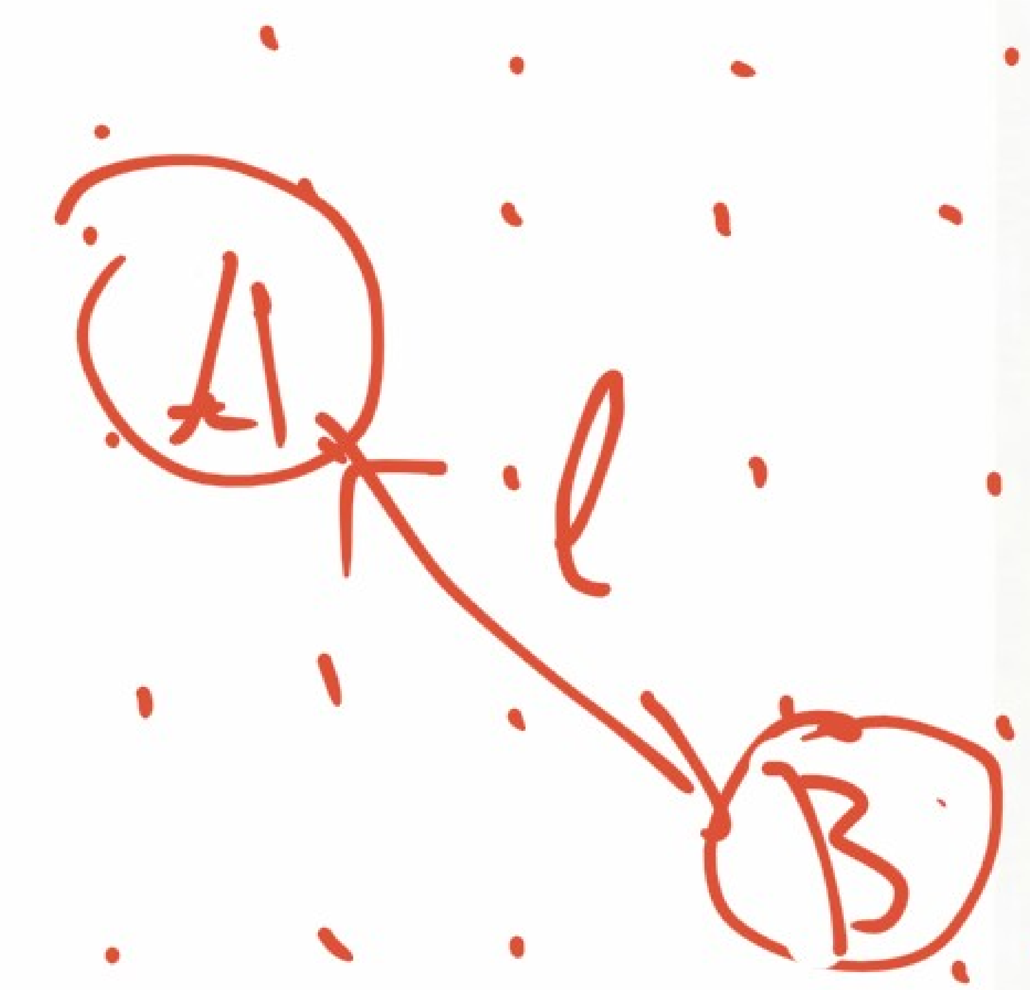
$\sqrt{N}$

$N^{1+\alpha}$

$$\Downarrow$$

$$\sup |F(E) - G(E)| \le C \frac{\ln^{2d} N}{s^3 \sqrt{N}}$$

$N \to \infty$



$X \in H_A$

$Y \in H_B$

$\sigma(H) = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \sim \sqrt{N}$

$$y_i = \frac{X_1 + \dots + X_N}{\sqrt{\sigma_1^2 + \dots + \sigma_N^2}}$$

$\boxed{\sigma_i} \neq \sigma_j$

$\langle X_j \rangle = 0$

$$\langle X \otimes Y \rangle - \langle X \rangle \langle Y \rangle \sim \left(\frac{3}{\ell}\right)^a$$

$a > 0$

- Translation invariance. ω_N

$\omega_N \quad \omega_{N+1}$

-

N

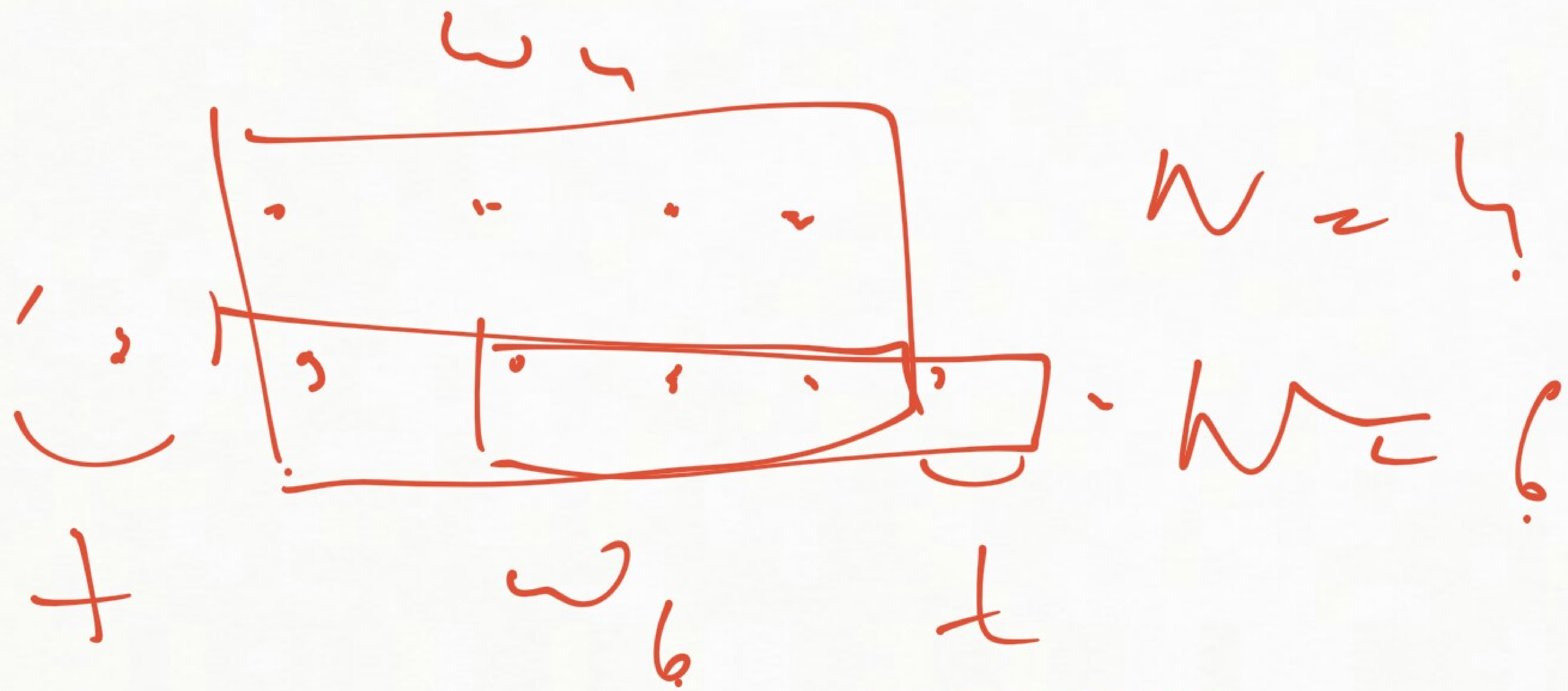
$\omega_N.$ $\quad N \to \infty$

$\lim_{N\to\infty} \omega_N.$

$\dim \mathcal{H}_a = d < \infty$

Araki

$$\boxed{\lim_{N\to\infty} \frac{\operatorname{tr}(\omega_N H_N)}{N} = u(\omega)}$$



$$\boxed{t\, 2_t\, \omega_6 = \omega_4}$$

$$\lim_{N\to\infty} \frac{-\mathrm{tr}\, \omega_N \ln \omega_N}{N} = \underline{s}(\omega)$$

$$S(\rho) = \mathrm{tr}(-\rho \cdot \ln \rho.) \qquad f(T) = \lim_{N\to\infty} \frac{F_N(T)}{N}$$

Variational principle.

$$\inf_{\omega} \left[u.(\omega) - T\, \underline{s}(\omega) \right] = f(\beta)$$

$$F = E - TS$$

F

E, β

$$E = \mathrm{tr}(\rho H)$$

S

$$E = \frac{\mathrm{tr}\, H e^{-\beta H}}{\mathrm{tr}\, e^{-\beta H}}$$

$$\arg\max \left[-\mathrm{tr}(\rho \ln \rho) \right] =$$

$$\boxed{E = \mathrm{tr}(\rho H)}$$

$$\beta = \beta(E)$$

$$= \frac{e^{-\beta H}}{Z}, \quad \boxed{\beta = \frac{1}{T}}$$

$$F = E - TS$$

$$\underline{F[\rho] = \operatorname{tr}(H\rho) + T \operatorname{tr}(\rho \ln \rho)}$$

$$\min_{\rho} F[\rho] = \boxed{F_\beta = -T \ln Z}$$

Muller, et al, CMP 340, 499 (2015)

$S = s(u)$

$\beta = \beta(u)$

$\boxed{E = uN}$

$$\underbrace{Q_N(u)}_{} = \#\left\{ \text{eigs of } H_N \leq uN \right\}$$

$$\boxed{\lim_{N\to\infty} \frac{\ln Q_N(u)}{N} = s(u)}$$

$s = s_N + O\left(\frac{1}{N}\right)$

$Q_N = \#\{E_n < uN\}$

$\ln Q_N = sN + o(N)$

$s = \lim_{N\to\infty} s_N$

$\Omega_N(E) = \frac{d\, Q_N(u)}{N\, du}$

$\lim_{N\to\infty} \frac{\ln Q_N}{N} = s(u)$

$\boxed{\Omega_N(E) = e^{N s_N(u) + O(1)}}$

$O(1)$ $N \gg 1$

$u - Ts(u)$ f u^A

$$q_N(E) = \frac{e^{-\beta E}}{Z_N}\,\Omega_N(E)$$

$$u_N(\beta) = \frac{1}{N}\,\frac{\mathrm{tr}\, H e^{-\beta H}}{\mathrm{tr}\, e^{-\beta H}}$$

$$u = u_N(\beta)$$

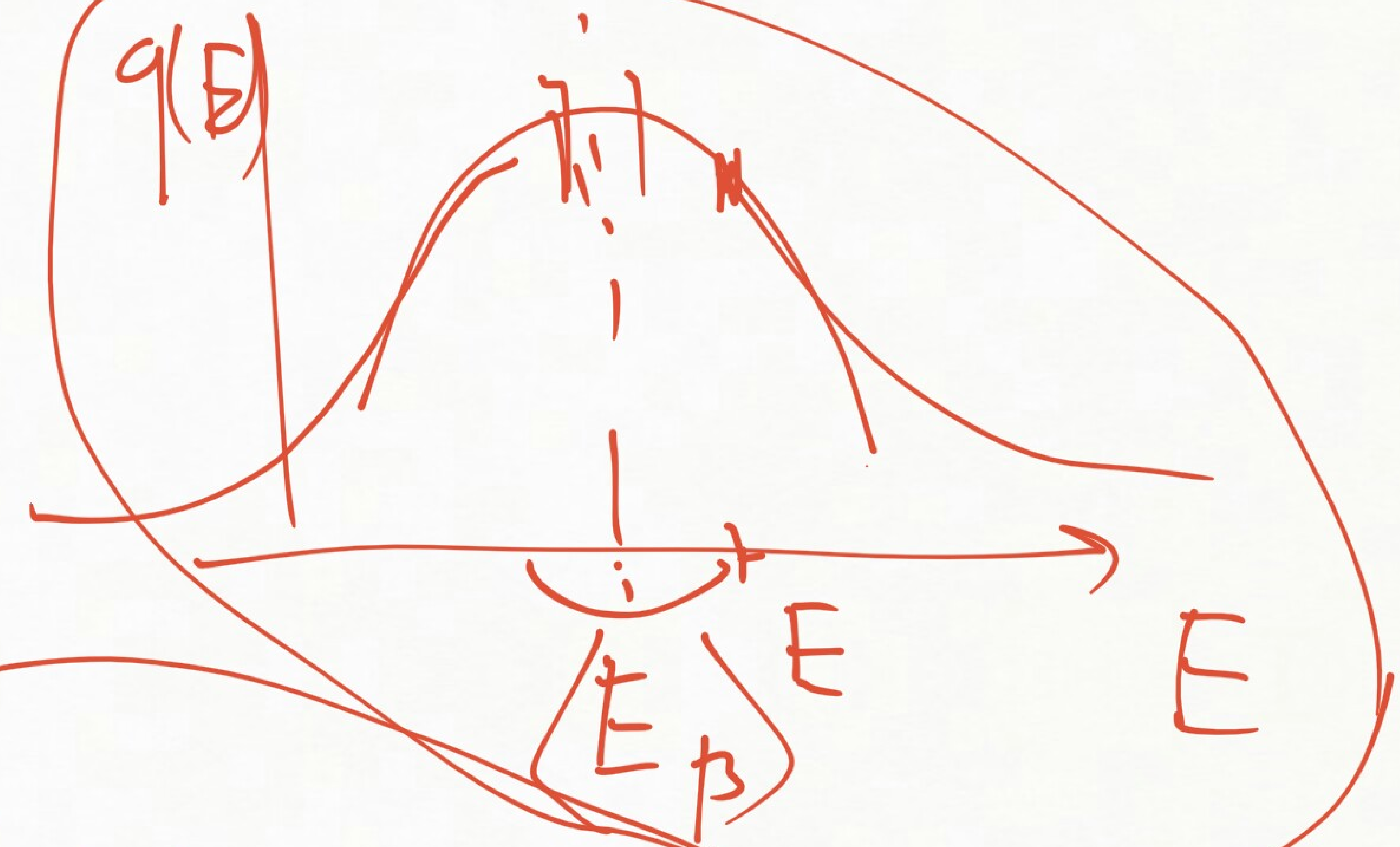
$$F_N = -T \ln Z_N = N f_N(\beta)$$

$$q_N(E) = e^{\beta N f_N - \beta N [u - T s_N(u)] + O(}$$

$u \to u + \varepsilon$

$u - T s(u)$ f_N

$q_N(E)$



$$f(u) = u - Ts(u)$$

$$u = u(T')$$
$$s(u) = s(T')$$

$$E - \langle E_\beta \rangle \sim N\delta$$

$$q(E) \sim e^{-N\beta\Delta f}$$

$\boxed{\alpha = 0}$

$\alpha > 0$

Critical systems

$$\boxed{g_N(E) \xrightarrow[N\to\infty]{} g(E)}$$

$$g(E) = e^{-\frac{(E-\langle H\rangle)^2}{2\sigma(H)}}$$

$$\sigma(H) \propto N$$

$$C(H) = \frac{dE}{dT}\frac{1}{N}$$

$$C \sim \sigma(H)$$

$$\sigma(H) \sim N \ln N$$

$$C_N \sim N \ln N$$

$$T_c$$

$$\alpha = 0$$

$$C(T) \sim \left| \ln |T - T_c| \right|$$

$$C_N \sim \ln N$$

$$C(T) \sim \left| \frac{T_c}{T - T_c} \right|^{\alpha}, \quad \alpha > 0$$

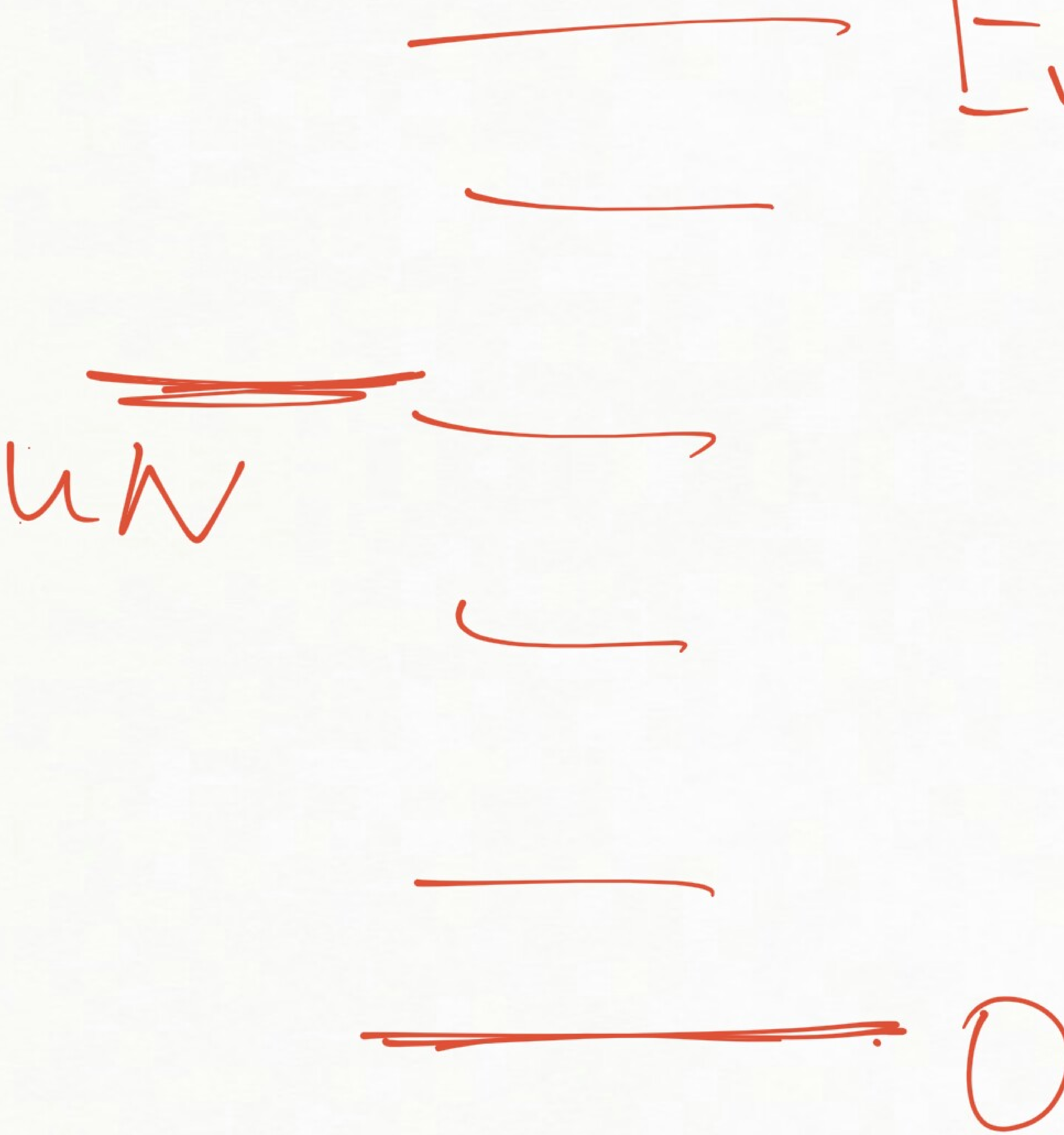
Standard $\sup_{E} |F_N(E) - G_N(E)| < \frac{\ln^{2d} N}{\sqrt{N}} \to 0$

$e^{-\frac{(u-\langle u\rangle)^2}{var(u)}}$

$q_N(uN) \xrightarrow{N\to\infty} g_N(uN) \quad \forall u \in [\ ]$

$|\,|\,|$ $\quad q_N - g_N \to 0 \quad N \to \infty$

$g_N \to g(u)$



$N \gg 1$

$E_{max} \sim N \frac{B_h}{N}$, $u \to \{$

$u = 0 \longrightarrow T = 0$

$u = \frac{B_h}{N} \longrightarrow T = \infty$

$0 < T < \infty$

$[u_{min}, u_{max}]$

(1) — $\langle X, Y \rangle \sim e^{-\ell/\xi}$

(2) — $\langle X, Y \rangle \sim \left(\frac{\xi}{\ell}\right)^{?}$

$C(T)$

(1) $\frac{C}{N} \sim O(1)$

(2) $\frac{C}{N} \sim \begin{cases} \ln N & \alpha < 0 \\ N^{a} & \alpha > 0 \end{cases}$