Motivation 1: $\frac{\partial S}{\partial \lambda} = \int d^2x \sqrt{\gamma}\, \underbrace{T\bar{T}}$

$$S = S_0 + \lambda \int d^2x \sqrt{\gamma}\, T\bar{T}$$

$$+ \ldots$$

$$E(\lambda) = \frac{1}{4\lambda}\left(1 - \sqrt{1 - 8\lambda E_0 + 16\lambda^2 J_0^2}\right)$$

Motivation 2: Wilsonian universality

$$S = \int d^4x \left[\partial_\mu\phi\, \partial^\mu\phi + V(\phi)\right]$$

$$[\phi] = 1$$

$$V(\phi) = \phi^2, \phi^3, \ldots, \phi^n$$

almost all are irrelevant

$\phi^5, \phi^6, \ldots \nearrow$

IR is universal

QM: $\int d\tau \left[(\partial_\tau q)^2 + V(q)\right]$

$[q] = -1/2$; interactions are relevant!!

<u>UV is universal</u>

<u>How do you modify the UV?</u>

$H \to f(H, Q_i)$; e.g. $f(H, L^2, L_z)$

$d>1$ QFT : $Q = \int d^{d-1}x \, J_0$

$$f(H) = H + \lambda H^2 + \ldots$$

nonlocal deformation $\rightsquigarrow \int d^{d-1}x \int d^{d-1}y \, J_0 J_0$

$d=1$. $\underline{H \to f(H)}$, $f(H)$ strictly monotonic

$$H|E\rangle = E|E\rangle$$

$$\Rightarrow f(H)|E\rangle = \underline{f(E)}|E\rangle$$

eigenvectors remain eigenvectors.

Finite-dimensional Hilbert space

$\hookrightarrow$ no new eigenvectors are created

$$H = -\sum J_{ijk} \psi_i \psi_j \psi_k \; ; \; \{\psi_i, \psi_j\} = \delta_{ij}$$

$f(H) = H + \lambda H^2$; matching as $\lambda \to 0$

$$[Q, H] = 0 \iff [Q, f(H)] = 0$$

$$E_0 = 0$$

$Z(\beta)$

$$Z(\beta)_f = \int_{-\infty}^{+\infty} dE \, e^{-\beta f(E)} \rho(E)$$

$$= \int_{-\infty}^{+\infty} dE \, e^{-\beta E} \rho_f(E)$$

$$\boxed{\rho_f(E) = \rho(f^{-1}(E)) \frac{df^{-1}(E)}{dE}}$$

$$K(\beta, \beta') = \frac{1}{\ } \int dE \, e^{-\beta f(E) + \beta' E}$$

$$K_f(\beta,\beta') = \frac{1}{2\pi i}\int_{C_0} dE\, e$$

$$\Rightarrow e^{-\beta f(E)} = \int_C d\beta'\, K_f(\beta,\beta')\, e^{-\beta' E}$$

$$\boxed{Z(\beta)_f = \int_C d\beta'\, Z(\beta')\, K_f(\beta,\beta')}$$

$$f(H) = H - 2\lambda H^2 \Rightarrow$$

$$K_f(\beta,\beta') = \frac{1}{\sqrt{8\pi\lambda\beta}} \exp\left(-\frac{(\beta-\beta')^2}{8\lambda\beta}\right)$$

$$\lambda < 0, \quad C = (0,\infty)$$

$$f(H) = \frac{1}{4\lambda}\left(1 - \sqrt{1-8\lambda H}\right)$$

$$\Rightarrow K_f(\beta,\beta') = \frac{\beta}{(\beta')^{3/2}\sqrt{-8\pi\lambda}} \exp\left(\frac{(\beta-\beta')^2}{8\lambda\beta'}\right)$$

$$\lambda < 0, \quad C = \gamma + ix, \quad x \in (-\infty,+\infty)$$

$$Z(\beta)_f = \left(\sum_{i=0}^{\infty} \frac{\beta^i(-f(-\partial_\beta) - \partial_\beta)^i}{i!}\right) Z(\beta)$$

1-pt. function

$$\langle \mathcal{O} \rangle_\beta \equiv \sum e^{-\beta E} \langle E | \mathcal{O} | E \rangle$$

$$\longrightarrow \sum e^{-\beta f(E)} \langle E | \mathcal{O} | E \rangle$$

$$\langle \mathcal{O} \rangle_\beta^{(f)} = \int d\beta' K(\beta, \beta') \langle \mathcal{O} \rangle_\beta$$

Vacuum 2-pt. fcn

$$\langle 0 | \mathcal{O}(\tau) \mathcal{O}(0) | 0 \rangle$$

$$= \langle 0 | e^{-H\tau} \mathcal{O}(0) e^{+H\tau} \sum |E\rangle\langle E| \mathcal{O}(0) |0\rangle$$

$$= \sum e^{-E\tau} |\langle 0 | \mathcal{O} | E \rangle|^2$$

$$\langle 0 | \mathcal{O}(\tau) \mathcal{O}(0) | 0 \rangle^{(f)} = \int d\tau' K(\tau, \tau') \langle \cdots \rangle$$

$$\langle \mathcal{O}(\tau) \mathcal{O}(0) \rangle = \frac{1}{\tau^{2\Delta}}$$

$$\rightsquigarrow \sim K_{2\Delta + 1/2}(-\tau/4\lambda)$$

Thermal n-pt. function

$$\sum_{E_1} e^{-\beta E_1} \langle E_1 | \mathcal{O}(\tau_1) \cdots \mathcal{O}(\tau_{n-1}) \mathcal{O}(0) | E_1 \rangle$$

$$= \sum_{E_1} \cdots \sum_{E_n} \langle E_1 | \mathcal{O} | E_2 \rangle \cdots \langle E_n | \mathcal{O} | E_1 \rangle \cdot \exp\left(-\sum_{i=0}^{n-1} \beta_i E_{i+1}\right)$$

$$\beta_i = \tau_i - \tau_{i+1} \quad \text{for } i = 0, \dots, n-2$$

$$\tau_0 = \beta, \quad \beta_{n-1} = \tau_{n-1}$$

$$e^{-\tau_1 (E_2 - E_1)} \quad e^{-\tau_2 (E_3 - E_2)}$$

$$E_2 - E_1 \to f(E_2 - E_1)$$

Applications

Worldlines

$$f(H) = \frac{1}{4\lambda}\left(1 - \sqrt{1 - 8\lambda\left(\sum p_i^2/2 + V(q_1, \dots, q_N)\right)}\right)$$

$$\mathcal{L}_E = \frac{1}{4\lambda}\left(1 - \sqrt{(1 - 4\lambda \dot{q}_i^2)(1 - 8\lambda V)}\right)$$

$$S_E = \frac{1}{4\lambda} \int d\tau \left(1 - \sqrt{g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}\right)$$

$$g_{\mu\nu} = \delta_{\mu\nu} (1 - 8\lambda V)$$

$$X^0(\tau) = \tau, \quad X^i(\tau) = 2\sqrt{-\lambda}\, q_i(\tau)$$

$\rightsquigarrow$ worldline action for 1d particle in $(N+1)$-dimensional space

$$Z(\beta)_f = \int \frac{[\mathcal{D}e][\mathcal{D}X][\mathcal{D}\Phi]}{\mathrm{Vol(Diff)}} e^{-S_0[e,\Phi]}$$

$$e^{-S[e;X;\lambda]}$$

Coupling systems

$$f(H) = H_1 + H_2 + \lambda H_1 H_2$$

phases the system

CPHASE gate

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ \vdots \end{pmatrix}$$

$$\begin{pmatrix} \ddots & \\ & e^{i\lambda} \end{pmatrix} \begin{pmatrix} |10\rangle \\ |11\rangle \end{pmatrix}$$

$e^{i\lambda H_1 H_2 t}$ for $t = 1$; $H_1 = H_2 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$

New large-N limit

SYK: $H = -\sum J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$

$$\langle J_{ijkl}^2 \rangle = \frac{6J^2}{N^3}, \quad \langle J_{ijkl} \rangle = 0$$

$$\langle Z \rangle = \int dJ_{ijkl} \exp\left(-\frac{N^3}{12J^2} J_{ijkl}^2\right) Z(J_{ijkl})$$

$$G(\tau, \tau') = \sum_{i=1}^{N} \langle \chi_i(\tau)\, \chi_i(\tau') \rangle$$

$H \to H + \lambda H^2$, need $\lambda \sim N^{-1}$ to preserve large-N limit

$$S = \int d^3x \left(\partial_\mu \phi_i \partial^\mu \phi_i + \lambda (\phi_i^2)^2 \right)$$

$\lambda \sim N^{-1}$

Scale $\lambda \sim N^0$, can't compute deformed correlator with standard large N

$H \to f(H)$

$H + \lambda H^2 + \dots$

Ising $H = -\sum J_{ij} \sigma_i \sigma_j \rightsquigarrow H + \lambda H^2$

$(\sum J_{ij} \sigma_i \sigma_j)^2$

$\uparrow \downarrow \uparrow \uparrow \cdots \uparrow \downarrow$