D=2 supergravities and consistent Kaluza-Klein truncations

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with G. Bossard, G. Inverso, A. Kleinschmidt, and H. Samtleben.

IFS seminar

8th of June 2023

D=2 ??

- D=2 gravity models are usually simpler than in higher-dimensions.

Einstein-Hilbert $S_{\rm EH} \sim \int d^2x \sqrt{-g}\, R = \text{total derivative}$

Simplest non-trivial models:
Dilaton gravity $S \sim \int d^2x \sqrt{-g}\Big(\rho\, R - 2\, V(\rho)\Big)$

$V(\rho) \propto \rho$: Jackiw-Teitelboim model

and D=2 is the lowest possible dimension for:

→ Riemann curvature

→ Black holes

→ Boundaries with dynamics

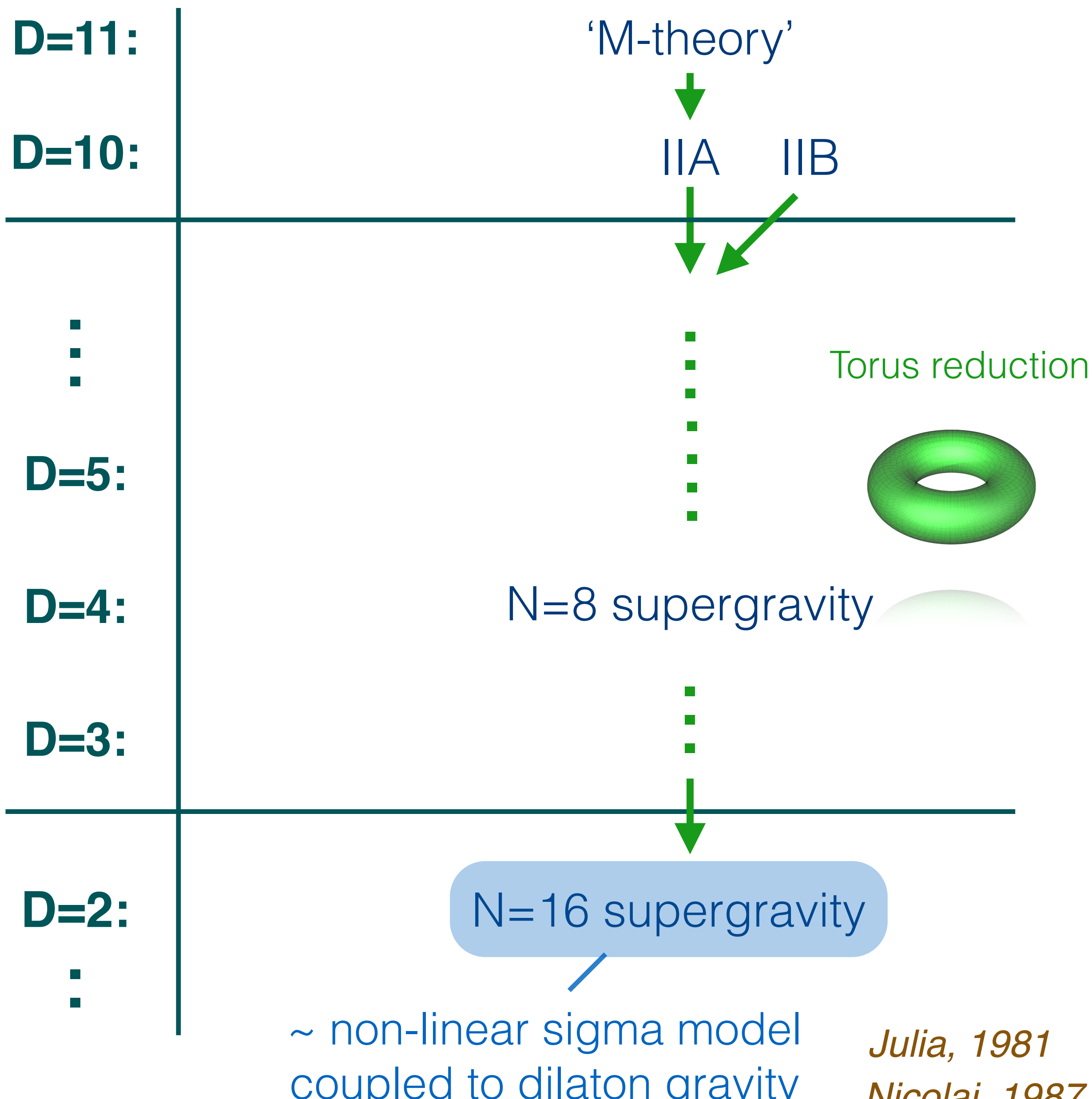
- There are also models which are much harder to study than in higher-dimensions…

In this talk: we are interested in **D=2 maximal supergravities**.

(in particular, those that arise from flux compactifications of `string theory').

Applications: holographic duality between string theory on AdS_2 backgrounds and various matrix quantum mechanics.

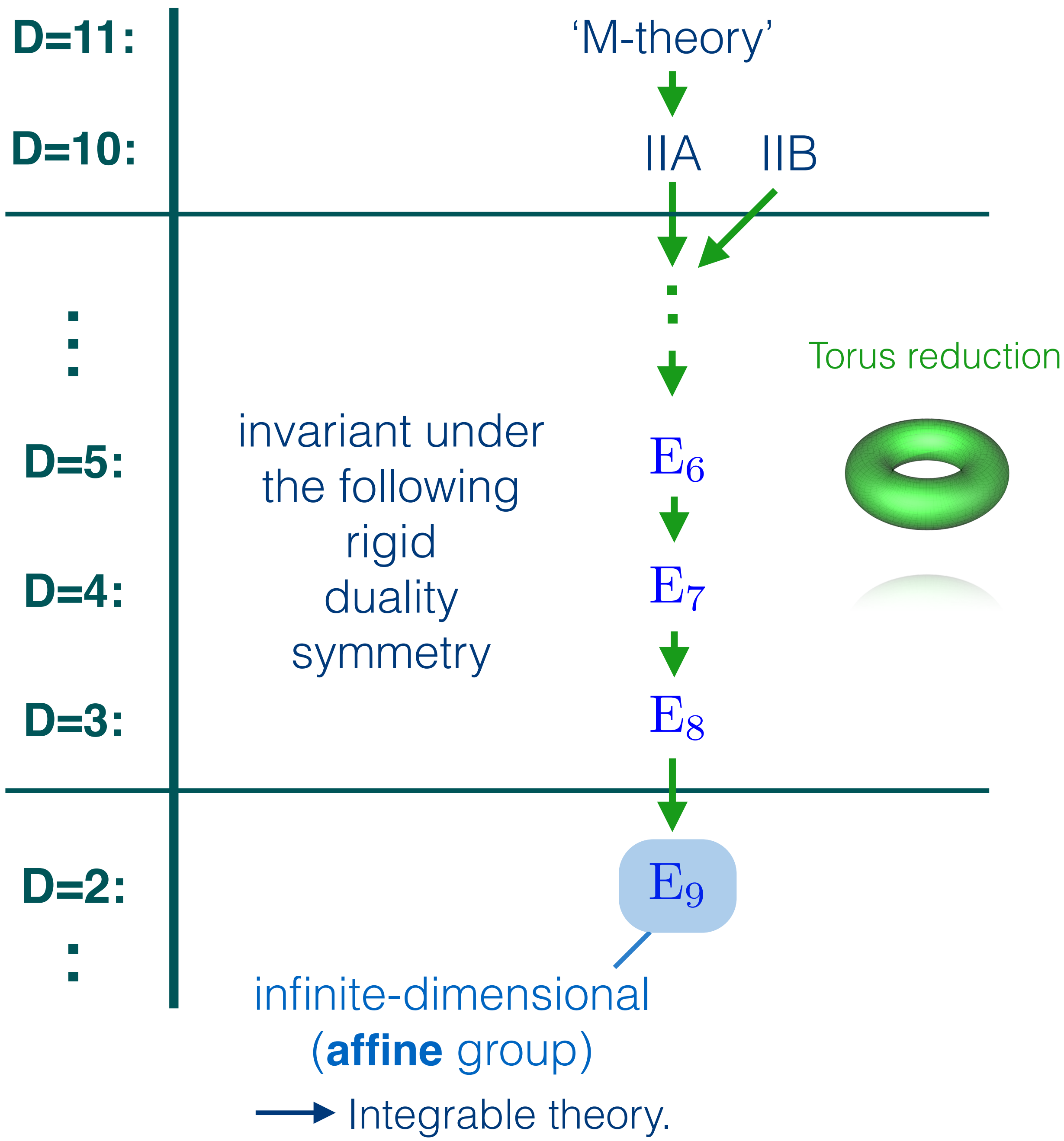
Origins of maximal D=2 supergravities



Julia, 1981
Nicolai, 1987
Breitenlohner, Maison, 1986

Ungauged supergravities

Origins of maximal D=2 supergravities



Ungauged supergravities

Origins of maximal D=2 supergravities

D=11: 'M-theory'

↓

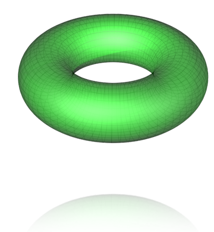
D=10: IIA IIB

↓

⋮

↓

Torus reduction



invariant under the following rigid duality symmetry

D=5: E_6

↓

D=4: E_7

↓

D=3: E_8

↓

D=2: E_9

⋮

infinite-dimensional (**affine** group)

⟶ Integrable theory.

Ungauged supergravities ⟶ **Gauged supergravities**

Gauging subgroups of the rigid duality symmetry group

"Duality covariant" description in terms of an embedding tensor

Nicolai, Samtleben, de Wit, Trigiante, Bergshoeff, ...

Origins of maximal D=2 supergravities

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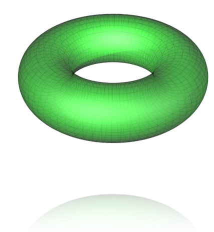

Example: $SO(8)$ gauged supergravity

De Wit, Nicolai, 1980's

Ungauged supergravities → **Gauged supergravities**

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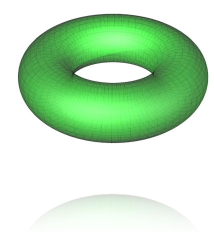

Kaluza-Klein truncation on S^7

Example: $SO(8)$ gauged supergravity

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Consistent Kaluza-Klein truncations:

- Allow to uplift all solutions
- Useful for holography

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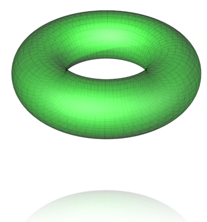

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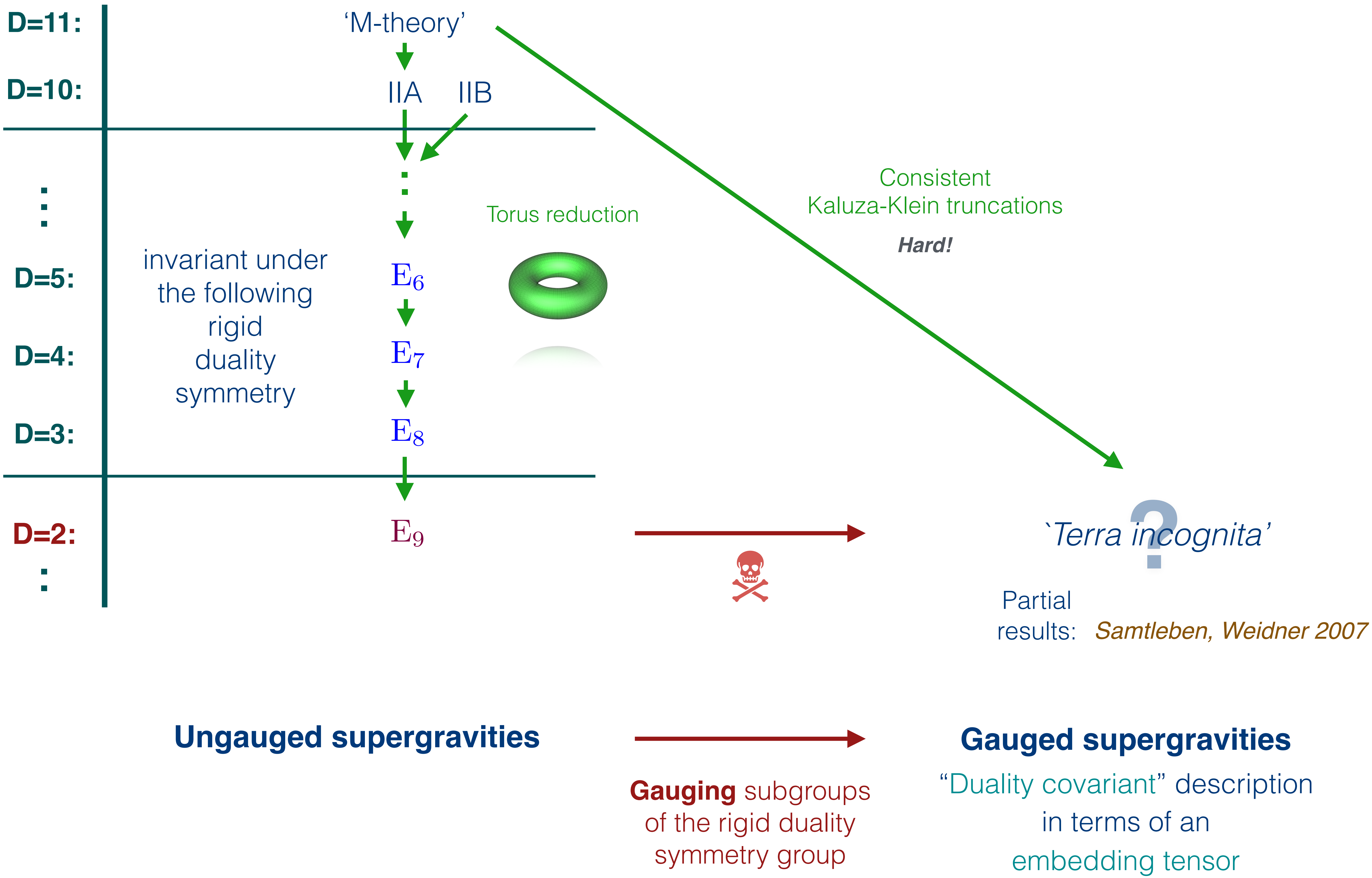
Consistent sphere truncations:

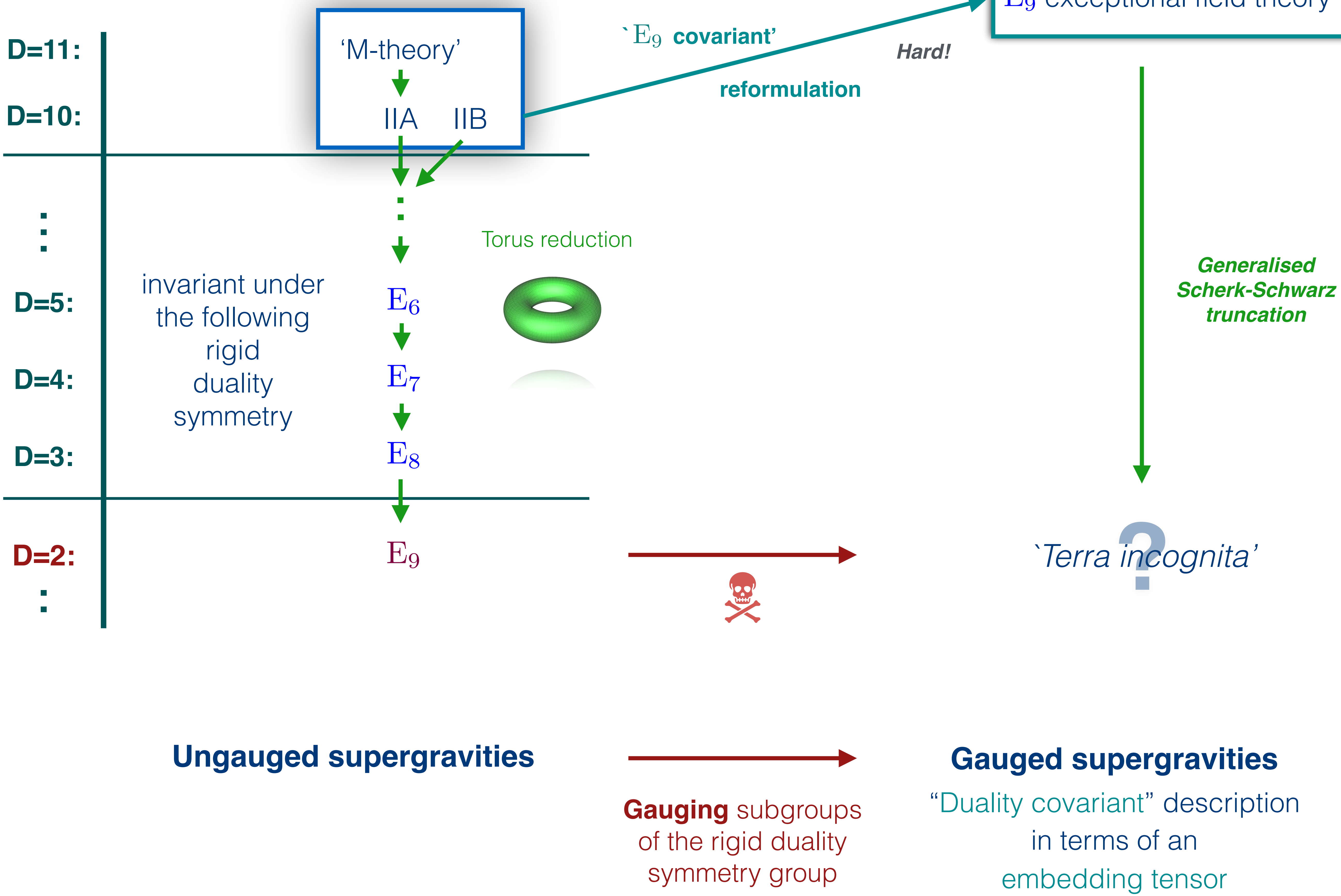
- D=11 SUGRA on S^7 — $AdS_4 \times S^7$ ⟷ M2-branes
- D=11 SUGRA on S^4 — $AdS_7 \times S^4$ ⟷ M5-branes
- D=10 IIB SUGRA on S^5 — $AdS_5 \times S^5$ ⟷ D3-branes

Prototypical examples of AdS/CFT

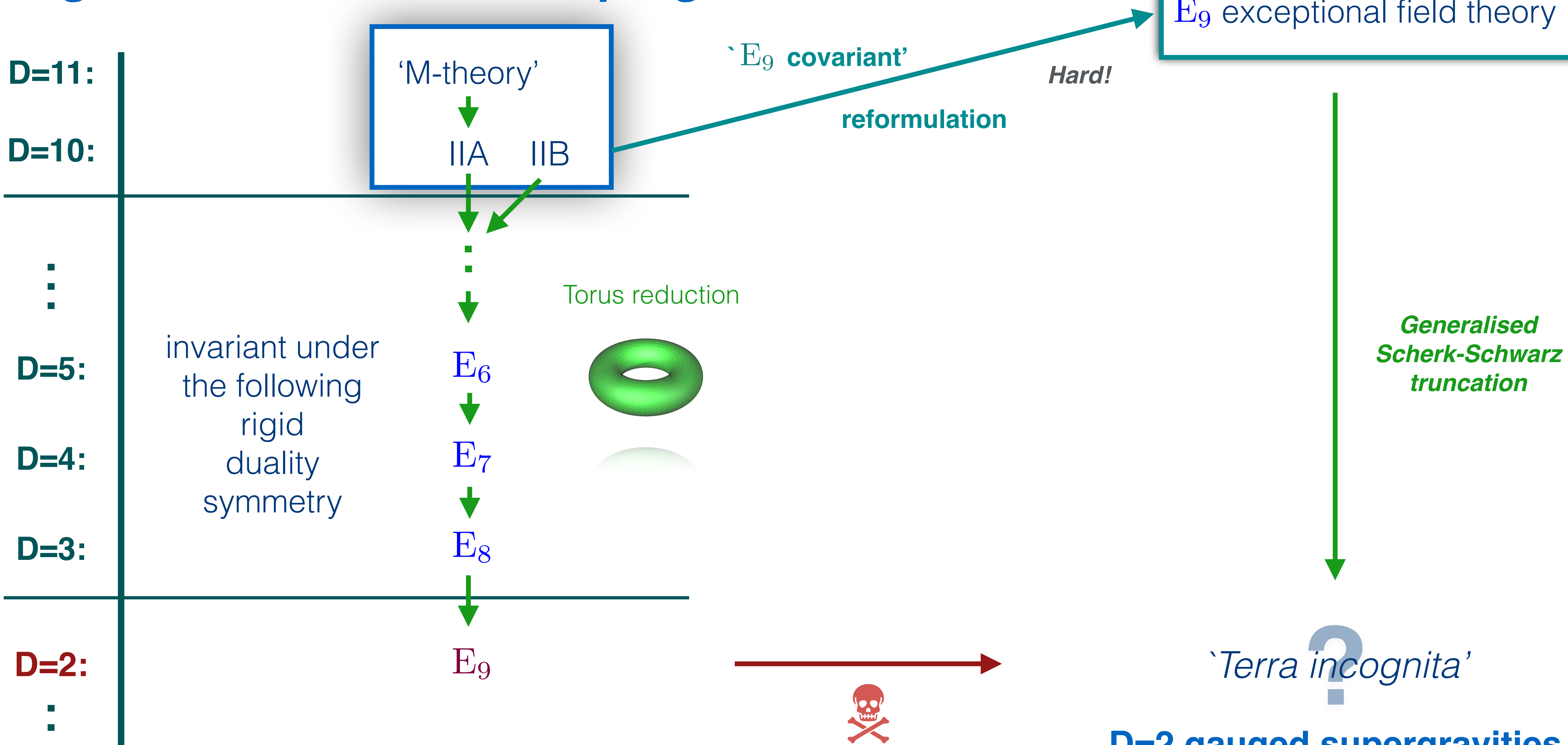
Origins of maximal D=2 supergravities



Origins of maximal D=2 supergravities



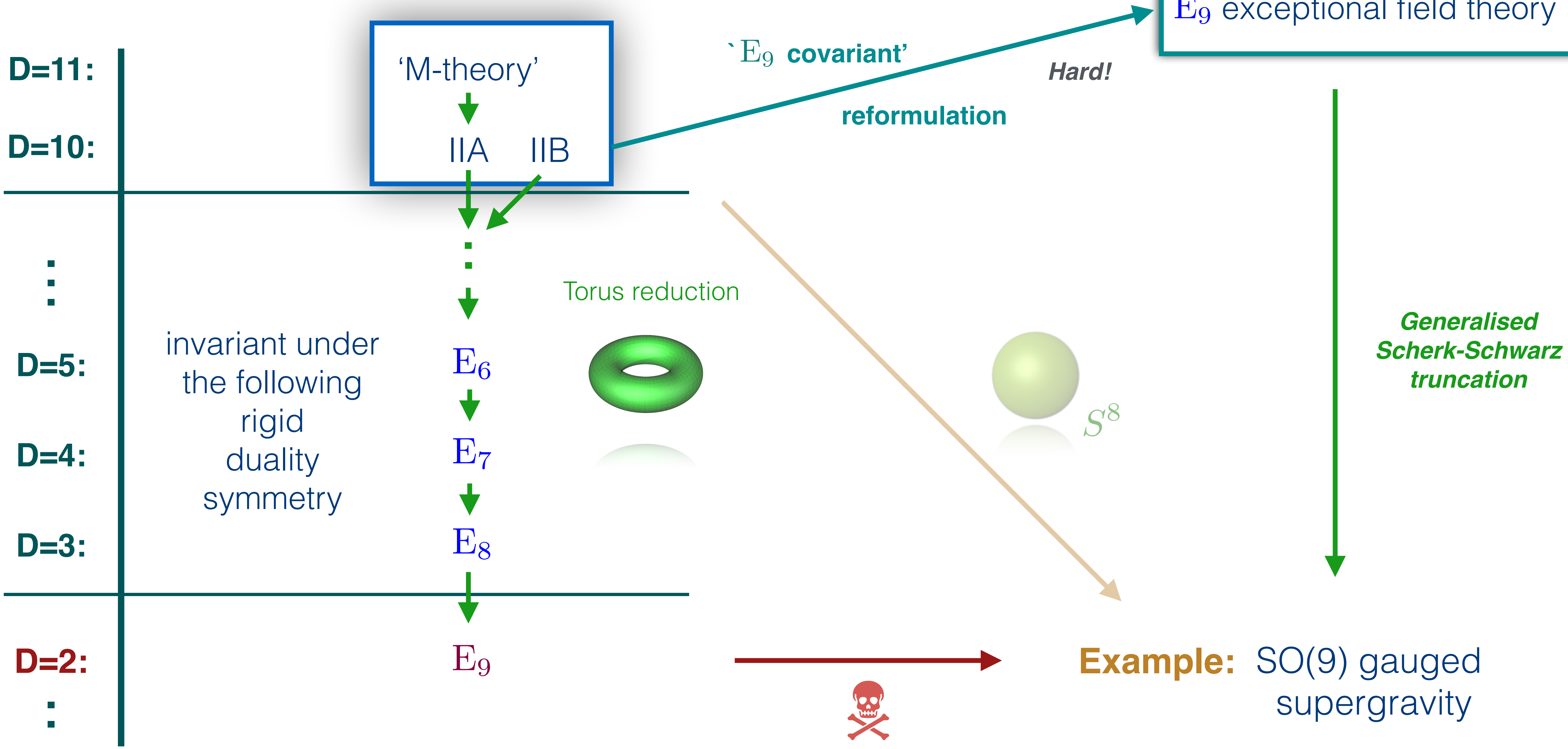
Origins of maximal D=2 supergravities



Results:

- Construction of all D=2 maximal gauged supergravities that admit a consistent embedding in D=10 or D=11 supergravity.

Origins of maximal D=2 supergravities



Results:

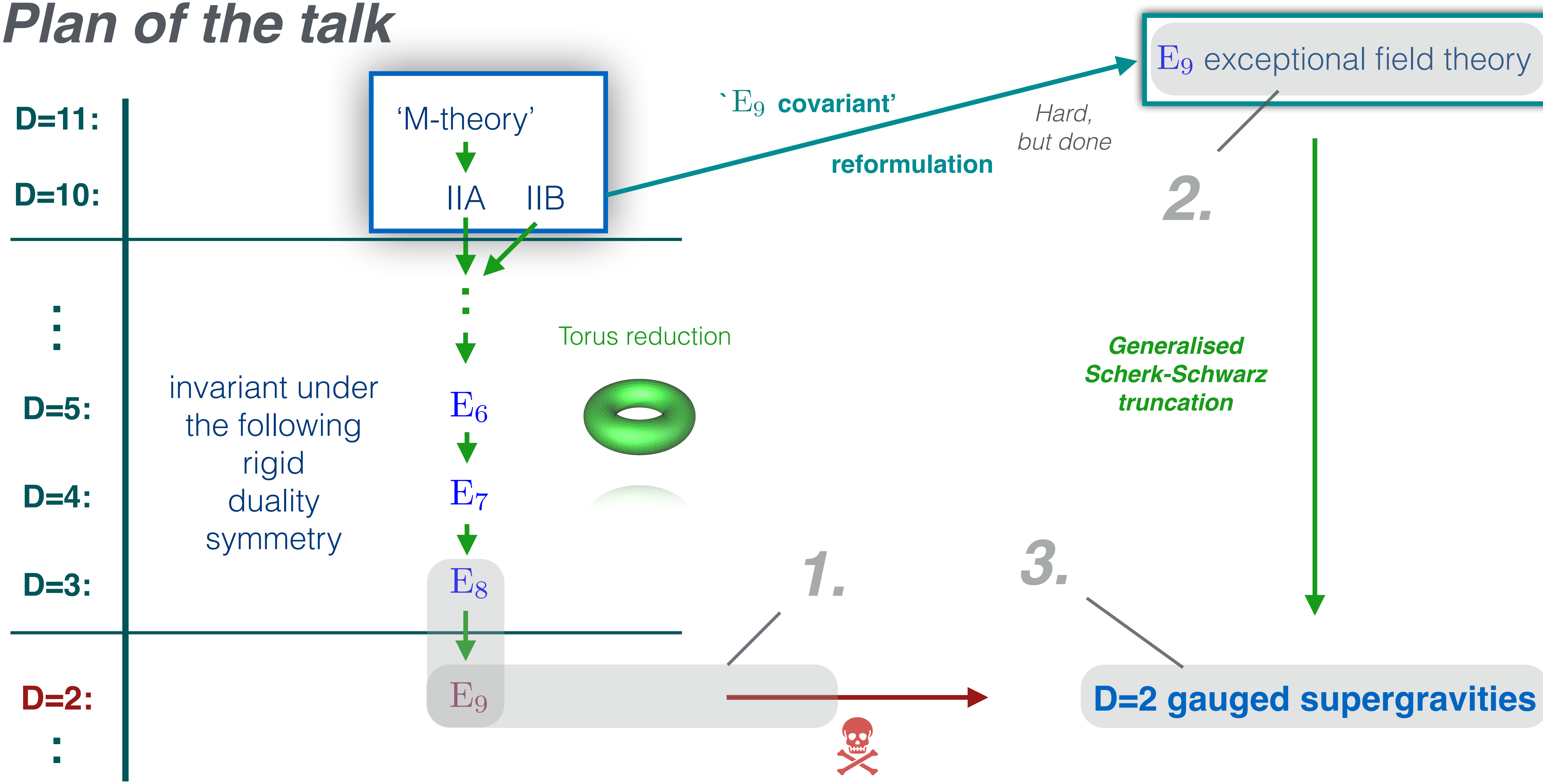
- Construction of all D=2 maximal gauged supergravities that admit a consistent embedding in D=10 or D=11 supergravity.

- Consistent truncation of IIA supergravity on S^8 to maximal SO(9) supergravity.

  → Holographic description of interacting D0-branes

Samtleben, Ortiz 2012

Plan of the talk



- Construction of all D=2 maximal gauged supergravities that admit a consistent embedding in D=10 or D=11 supergravity.
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Ungauged D=2 supergravity

$$\hat{\mu} = 1, 2, 3 = (\mu, 3)$$
$$\hat{a} = (a, 3)$$

Bosonic D=3 supergravity Lagrangian: *Marcus, Schwarz, 1983*

$$\mathcal{L}_{D=3} = \underbrace{\hat{e}}_{\det(\hat{e}_{\hat{\mu}}{}^{\hat{a}})}\, R^{(3)} - \hat{e}\, \mathrm{tr}\big(P_{\hat{\mu}}\, P^{\hat{\mu}}\big)$$

$E_8/SO(16)$

sigma model

Maurer Cartan form : $\underbrace{(\partial_{\hat{\mu}} V)V^{-1}}_{\in\, \mathfrak{e}_8} = P_{\hat{\mu}} + \underbrace{Q_{\hat{\mu}}}_{\in\, \mathfrak{so}(16)}$

$\mathrm{E}_8/\mathrm{SO}(16)$: parametrized by 128 physical scalars

Rigid symmetry : E_8

zweibein KK vector

Using the $\mathrm{SO}(2,1)$ Lorentz symmetry, fix the dreibein to: $\hat{e}_{\hat{\mu}}{}^{\hat{a}} = \begin{pmatrix} e_\mu{}^a & \rho\, A_\mu \\ 0 & \rho \end{pmatrix}$,

dilaton

and perform a (circle) reduction to 2 dimensions → All fields independent of x^3.

The *bosonic D=2 Lagrangian* then reads:

$$\mathcal{L}_{D=2} = e\, \rho\big[R^{(2)} - \cancel{\tfrac{1}{4}\rho^2 \underbrace{F_{\mu\nu}}_{= 2\, \partial_{[\mu} A_{\nu]}} F^{\mu\nu}} - \mathrm{tr}(P_\mu\, P^\mu)\big]$$

⚠ In D=2, the dilaton cannot be removed by a Weyl rescaling

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zweibein
KK vector
dilaton

and perform a (circle) reduction to 2 dimensions → All fields independent of x^3.

The *bosonic D=2 Lagrangian* then reads:

$$\mathcal{L}_{D=2} = 2\, \partial_\mu \sigma\, \partial^\mu \rho - \rho\, \mathrm{tr}(P_\mu\, P^\mu)$$

Conformal gauge
$e_\mu{}^\alpha = e^\sigma\, \delta_\mu^\alpha$

σ : conformal factor

Rigid symmetries: E_8 + Weyl rescaling $\sigma \xrightarrow[\mathrm{K}]{} \sigma + c$

On-shell symmetries in D=2

Consider the <u>field equations</u>:

- $\square\,\rho = \partial_\mu I^\mu_{(\rho)} = 0 \xrightarrow{\text{dualisation}} I^\mu_{(\rho)} = \partial^\mu \rho = \epsilon^{\mu\nu}\,\partial_\nu \tilde{\rho}$ $\qquad \underset{\text{duals}}{\rho \Longleftrightarrow \tilde{\rho}}$

  Free field

- $\partial_\mu(\rho \underbrace{V^{-1} P^\mu V}_{\in\, \mathfrak{e}_8}) = \partial_\mu I^\mu_{(1)} = 0 \xrightarrow{\text{dualisation}} I^\mu_{(1)} = \rho\, V^{-1} P^\mu V = \epsilon^{\mu\nu}\,\partial_\nu Y_1$

$$\longrightarrow \quad I^\mu_{(2)} = (\rho\,\tilde{\rho}\,\delta^\mu_\nu + \epsilon^{\mu\lambda}\,\eta_{\lambda\nu}\,\rho^2) V^{-1} P^\nu V - \tfrac{1}{2}\big[Y_1\,, \partial^\mu Y_1\big] = \epsilon^{\mu\nu}\,\partial_\nu Y_2$$

$\longrightarrow \;\cdots\; \longrightarrow$ infinite tower of ($\mathfrak{e}_8$ -valued) dual scalar fields Y_n .

Integrability conditions given by the *conservation of the currents* $I^\mu_{(n)}$.

$\longrightarrow$ 'consistency of the tower' relies on the field equations.

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Extra rigid symmetries of the field equations:

- $\underset{\mathbb{R}_{L_{-1}}}{\tilde{\rho} \longrightarrow \tilde{\rho} + c}$

- $Y_n \longrightarrow Y_n + \underbrace{C_n}_{\in\,\mathfrak{e}_8}$ infinite number of shifts
- *Non-linearly realised ('hidden') symmetries...*

E_8 loop group

On-shell symmetries in D=2

Consider the field equations:

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Full **on-shell symmetry** of the theory: $E_9 \ltimes \mathbb{R}_{L_{-1}}$

includes: E_8 loop group $\qquad \tilde{\rho} \longrightarrow \tilde{\rho} + c$

Weyl rescaling

⋮

E_9 affine group

- Non-linear realisation: scalars parametrise coset space $\sim \dfrac{E_9 \ltimes \mathbb{R}_{L_{-1}}}{K(E_9)}$

On-shell symmetries in D=2

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$\longrightarrow$ ... $\longrightarrow$ infinite tower of ($\mathfrak{e}_8$ -valued) dual scalar fields Y_n .

How to collectively encode these duality relations?

- Use a generating function known as the *linear system* *Belinsky, Zakarov 1978* *Breitenlohner, Maison 1986*

or equivalently [...long story...]

- a ***twisted self-duality equation*** *Julia, Nicolai 1996* *Paulot 2004*

Requires to further enlarge the group theoretical structure: $\sim \dfrac{E_9 \rtimes \mathrm{Vir}^-}{K(E_9)}$

Allows to define fields
In lowest weight reps.

Affine algebra and representation

$A, B \ldots$ adjoint $\mathfrak{e}_8$ indices

- Start with the loop algebra over $\mathfrak{e}_8$, denoted $\tilde{\mathfrak{e}}_8$:

$$\underbrace{X}_{\in\, \tilde{\mathfrak{e}}_8} \,:\, S^1 \longrightarrow \mathfrak{e}_8 \xrightarrow{\text{Fourier expansion}} X(\theta) = \sum_{n\in\mathbb{Z}} X_{n\,a}\, \underbrace{t^A\, e^{2\pi i n\theta}}_{T_n^A \,:\, \textbf{loop generators}}$$

- Affine extension: $\hat{\mathfrak{e}}_8 = \langle \tilde{\mathfrak{e}}_8 \,,\, \mathrm{K} \rangle$

$$[T_n^A \,,\, T_m^B] = f^{AB}{}_C\, T_{n+m}^C + n\, \eta^{AB} \delta_{n,-m} \mathrm{K}$$

central charge

- Virasoro extension of the affine algebra $\hat{\mathfrak{e}}_8 \oplus \mathfrak{vir}$ Generators: $T^\alpha = \{T_n^A \,,\, \mathrm{K} \,,\, L_m\}$

via Sugawara construction

$$[T_n^A \,,\, L_m] = n\, T_{n+m}^A$$

Virasoro generators $m \in \mathbb{Z}$

$$\boxed{\mathfrak{e}_9 = \langle \hat{\mathfrak{e}}_8 \,,\, L_0 \rangle}$$

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$$\boxed{\mathfrak{e}_9 = \langle \hat{\mathfrak{e}}_8 \,,\, L_0 \rangle}$$

- We define shift operators S_n acting as

$$\boxed{S_n(T^A_m) = T^A_{n+m} \qquad S_n(L_m) = L_{n+m} \qquad S_n(\mathrm{K}) = 0}$$

Important : $[T^A_m \,, S_n(T^B_p)] = S_n\big([T^A_n \,, T^B_m]\big) + \ldots \mathrm{K} \longrightarrow$ *Not automorphisms of the affine algebra*

Affine algebra and representation

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central charge

- Relevant (faithful) representation: ***basic representation*** $R(\Lambda_0)$

Lowest weight representation $\quad T_0^A |0\rangle = 0 \qquad \mathrm{K}|0\rangle = |0\rangle \qquad T_n^A |0\rangle = 0\,, \text{ for all } n > 0$

$\mathfrak{e}_8$ invariant vacuum

We will use both Fock space and index notations:

$$|V\rangle = V^M |e_M\rangle$$

generic vector in $R(\Lambda_0)$

basis

In the basic representation, we also define a Hermitian conjugation:

$$(T_n^A)^\dagger = T_{-n}^A \qquad (L_n)^\dagger = L_{-n} \qquad (\mathrm{K})^\dagger = \mathrm{K}$$

Extended coset structure

Affine group

generated by $L_n \leq 0$

The (infinite number of) scalar fields parametrize the $\dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$ coset space.

Maximal unitary subgroup of E_9
whose elements H satisfy: $H^\dagger H = 1$

Twisted self-duality equation:

$\boxed{\star\mathcal{P} = S_1(\mathcal{P})}$ in terms of $\mathcal{P}_\mu = \frac{1}{2}(\partial_\mu \mathcal{V}\mathcal{V}^{-1} + \mathrm{h.c}) = \mathcal{P}_{\mu\,\alpha} \otimes T^\alpha$

Encodes all the D=2 dynamics. How..?

Extended coset structure

Affine group — generated by $L_n \leq 0$

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We can choose a Borel gauge, and write the coset representative as:

$\mathrm{E}_8/\mathrm{SO}(16)$ representative

$$\mathcal{V} = \underbrace{\rho^{-L_0} e^{-\phi_1 L_{-1}} e^{-\phi_2 L_{-2}} \ldots}_{\in \mathrm{Vir}^-} \underbrace{V e^{Y_{1A} T^A_{-1}} e^{Y_{2A} T^A_{-2}} \ldots e^{-\sigma \mathrm{K}}}_{\in \hat{E}_8}$$

Extra 'Virasoro scalars' $\phi_n\,, n \in \mathbb{N}^+$

$\mathfrak{vir}$ *components of twsd:*

$$\rightarrow \left| \begin{array}{l} d\phi_1 = 2 \star d\rho \\ d\phi_2 = d(\rho^2) \\ d\phi_3 - \phi_1 d\phi_2 = 2\rho^2 \star d\rho \\ \vdots \end{array} \right.$$

Can be integrated to $\longrightarrow$

$$\begin{array}{l} \phi_1 = 2\tilde{\rho} \\ \phi_2 = \rho^2 \\ \phi_3 = 2\rho^2 \tilde{\rho} \\ \vdots \end{array}$$

Only dual fields left are $\rho, \tilde{\rho}$ *:* $d\tilde{\rho} = \star d\rho$

Loop components of twsd:

$$\rightarrow \left| \begin{array}{l} dY_1 = \star(\rho V^{-1} P V) \\ \vdots \end{array} \right.$$

Recover the non-linear relations between all the dual scalars

Extended coset structure

The (infinite number of) scalar fields parametrize the $\dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$ coset space.

Twisted self-duality equation:

$$\boxed{\star\mathcal{P} = S_1\big(\mathcal{P}\big)}$$

Not covariant...

in terms of $\mathcal{P}_\mu = \frac{1}{2}(\partial_\mu \mathcal{V}\mathcal{V}^{-1} + \mathrm{h.c}) = \mathcal{P}_{\mu\,\alpha} \otimes T^\alpha$

covariance: $\delta_\lambda^{K(E_9)} \mathcal{P}_\mu = [\lambda, \mathcal{P}_\mu]$

Extended coset structure

The (infinite number of) scalar fields parametrize the $\dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$ coset space.

Twisted self-duality equation:

$$\boxed{\star\mathcal{P} = \mathcal{P}^{(1)}}$$

in terms of $\mathcal{P}_\mu = \frac{1}{2}(\partial_\mu \mathcal{V}\mathcal{V}^{-1} + \mathrm{h.c}) = \mathcal{P}_{\mu\,\alpha} \otimes T^\alpha$

covariance: $\delta_\lambda^{K(E_9)} \mathcal{P}_\mu = [\lambda, \mathcal{P}_\mu]$

Covariance of the duality equation requires to introduce an extra one-form field χ_μ :

Transforms together with $\mathcal{P}_{\mu\,\alpha}$ in an **indecomposable representation**

$$\begin{pmatrix} \mathcal{P}_{\mu\,\alpha} \\ \chi_\mu \end{pmatrix} \xrightarrow[K(E_9)]{} \begin{pmatrix} \bullet & 0 \\ \bullet & \bullet \end{pmatrix} \begin{pmatrix} \mathcal{P}_{\mu\,\alpha} \\ \chi_\mu \end{pmatrix}$$

group 1-cocycle

Cannot be diagonalised

Used to build covariant shifted one-forms: $\mathcal{P}_\mu^{(1)} = \mathcal{P}_{\mu\,\alpha} \otimes S_1(T^\alpha) + \chi_\mu \otimes \mathrm{K}$

compensate the lack of covariance

No new on-shell degrees of freedom: $\star\mathcal{P}_{\mathrm{K}} = \chi$

Pseudo-Lagrangian for ungauged SUGRA?

Duality covariant dynamics encoded in

$$L = \overset{?}{L}^{\text{kin.}} + \overset{?}{L}^{\text{top.}} \quad + \quad \star\mathcal{P} = \mathcal{P}^{(1)}$$

→ Involve all the dual fields $\mathcal{V} \in \dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$

→ Relevant to study covariant deformations (embedding tensor)

- <u>Kinetic term:</u> natural/naive guess doesn't work $L_{\text{kin.}} \sim \eta^{\alpha\beta}\mathcal{P}_{\mu\,\alpha}\mathcal{P}^{\mu}{}_{\beta} \longrightarrow$ no invariant bilinear form over $\hat{\mathfrak{e}}_8 \oplus \mathfrak{vir}$

Pseudo-Lagrangian for ungauged SUGRA?

$$L = \cancel{L}^{\text{kin.}} + ? L^{\text{top.}} \quad + \quad \star\mathcal{P} = \mathcal{P}^{(1)}$$

Duality covariant dynamics encoded in

- Involve all the dual fields $\mathcal{V} \in \dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$
- Relevant to study covariant deformations (embedding tensor)

- Topological term:

Start with **Maurer-Cartan equation:** $\underbrace{\big(d - \delta_{\mathcal{Q}}^{K(\mathfrak{e}_9)}\big)}_{K(\mathrm{E}_9)\ \textit{covariant derivative}}\mathcal{P} = 0$

$= d\mathcal{P} - [\mathcal{Q}, \mathcal{P}]$

covariance: $\delta_\lambda^{K(E_9)}\mathcal{P}_\mu = [\lambda, \mathcal{P}_\mu]$

$K(\mathfrak{e}_9)$ covariant

Shift: $S_1\Big(\big(d - \delta_{\mathcal{Q}}^{K(\mathfrak{e}_9)}\big)\mathcal{P}\Big) \xrightarrow[\text{under } K(\mathfrak{e}_9)]{\text{covariantize}} S_1\Big(\big(d - \delta_{\mathcal{Q}}^{K(\mathfrak{e}_9)}\big)\mathcal{P}\Big) + \underbrace{\big(d - \delta_{\mathcal{Q}}^{K(\mathfrak{e}_9)}\big)\chi\ \mathrm{K}}_{\textbf{Invariant top form}}$

central charge

Pseudo-Lagrangian for ungauged SUGRA?

Duality covariant dynamics encoded in

$$L = L^{\text{top.}} \quad + \quad \star\mathcal{P} = \mathcal{P}^{(1)}$$

- Involve all the dual fields $\mathcal{V} \in \dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$
- Relevant to study covariant deformations (embedding tensor)

- Topological term:

$$L^{\text{top.}} = \rho\Big(d\chi_1 - \eta^{AB} \sum_{n\in\mathbb{Z}} n\, \mathcal{Q}^n_A \mathcal{P}_B^{-n-1} + \ldots\Big)$$

includes a Wess-Zumino term

$\mathfrak{e}_8$ Killing form

Loop components of $\mathcal{P}_{\mu\,\alpha}$.

after non-trivial manipulations, one can show that:

$$L^{\text{top.}} = \underbrace{\text{physical action}}_{\text{Only depends on physical scalars}} + (\text{twisted sd})^2$$

$\rho\,,\ \sigma\,,\ \mathcal{P}^0_A$

E_8 scalars

How to turn on gaugings?

Pseudo-Lagrangian for gauged SUGRA?

Duality covariant dynamics encoded in

$$L_{\text{gsugra}} = L^{\text{top.}}_{\text{gsugra}} - \star V_{\text{gsugra}} \quad + \quad \star\mathcal{P} = \mathcal{P}^{(1)}$$

→ Involve all the dual fields $\mathcal{V} \in \dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$

→ Introduce gauge fields $|\mathcal{A}\rangle$: $\mathcal{D}_\mu = \partial_\mu - g\,\mathcal{A}_\mu{}^M\,\Theta_{M,\alpha}\,T^\alpha$

Gauge covariant currents $\mathcal{P}_\mu = \dfrac{1}{2}(\mathcal{D}_\mu \mathcal{V}\,\mathcal{V}^{-1} + \mathrm{h.c})$

$\langle\Theta_\alpha|$: embedding tensor

selects the gauge group

$G \subset \hat{E}_8 \rtimes \mathrm{Vir}^-$

Must satisfy algebraic and
'representation' constraints

Pseudo-Lagrangian for gauged SUGRA?

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selects the gauge group

$G \subset \hat{E}_8 \rtimes \text{Vir}^-$

- Topological term:

Gauged Maurer-Cartan equation:

$$(\mathcal{D} - \delta_{\mathcal{Q}}^{K(\mathfrak{e}_9)})\mathcal{P} = g\langle\Theta_\alpha|\mathcal{F}\rangle\,(\mathcal{V}T^\alpha\mathcal{V}^{-1} + \text{h.c})$$

Field strength $\mathcal{F}^M_{\mu\nu}$

Same trick
(shift, then covariantize)

→

$$L^{\text{top.}}_{\text{gsugra}} = \rho(\mathcal{D}\chi_1 - \eta^{AB}\sum_{n\in\mathbb{Z}} n\,\mathcal{Q}^n_A \mathcal{P}^{-n-1}_B \; + \ldots\,) + \ldots$$

$$= \rho\,\mathbf{D}\chi \; + \mathcal{O}\Big(\mathcal{V}, \langle\Theta_\alpha|, |\mathcal{F}\rangle\Big)$$

Pseudo-Lagrangian for gauged SUGRA?

Duality covariant dynamics encoded in

$$L_{\text{gsugra}} = L^{\text{top.}}_{\text{gsugra}} - \star V_{\text{gsugra}} \quad + \quad \star\mathcal{P} = \mathcal{P}^{(1)}$$

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- Topological term: $L^{\text{top.}}_{\text{gsugra}} = \rho\,\mathbf{D}\chi_1 + \mathcal{O}\Big(\mathcal{V}, \langle\Theta_\alpha|, |\mathcal{F}\rangle\Big)$

- Scalar potential V_{gsugra} : fixed by SUSY. Problematic in D=2...**?**

?
Yukawa
couplings

No trick... ⟶ ***Get it from Kaluza-Klein truncations***

$\hat{E}_8 \rtimes \mathrm{Vir}^-$ *exceptional field theory (ExFT)*

Reformulation of higher-dimensional (11D/IIB) supergravities which makes the duality symmetry of D=2 maximal supergravities manifest.

- Shares most of the on-shell field content of D=2 maximal supergravities:

$$\{\mathcal{V}\,,\,\chi_\mu\,,\,\mathcal{A}_\mu{}^M\,,\,\ldots\}$$

$$\frac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$$

- ***But*** all fields and gauge parameters depend on the coordinates: $(x^\mu\,,\,y^M)$

D=2 external spacetime

internal space

Kaluza-Klein split

External derivative: ∂_μ Internal derivative: ∂_M

$\hat{E}_8 \rtimes \mathrm{Vir}^-$ *exceptional field theory (ExFT)*

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  $\mathcal{V} \in \dfrac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(E_9)}$

- ***But*** all fields and gauge parameters depend on the coordinates: $(x^\mu\,,\,y^M)$

  External derivative: ∂_μ Internal derivative: ∂_M or $\langle\partial|$

  x^μ: D=2 external spacetime; y^M: internal space

  Kaluza-Klein split

- The dynamics is *fully fixed by the bosonic gauge symmetries* of ExFT

  → generalised diffeomorphisms with parameters $\Lambda^M(x,y)\,,\ldots$

Generalised Lie derivative

$$\mathcal{L}_{|\Lambda\rangle\ldots}\underbrace{\Phi}_{\text{Generic field}} = \underbrace{\Lambda^M\partial_M\Phi}_{\text{Transport term}} + (\underbrace{\eta_{0\,\alpha\beta}\,T^{\alpha\,M}{}_N\,\partial_M\Lambda^N}_{\mathfrak{e}_9\ \text{rotation term}} + \underbrace{\ldots}_{\mathfrak{vir}^-\ \text{rotation}})\,\delta^\beta\Phi$$

$\hat{E}_8 \rtimes \mathrm{Vir}^-$ *exceptional field theory (ExFT)*

Generalised Lie derivative

$$\mathcal{L}_{|\Lambda\rangle\,\ldots}\,\Phi = \Phi\langle\partial|\Lambda\rangle + \left(\eta_{0\,\alpha\beta}\langle\partial|T^\alpha|\Lambda\rangle + \ldots\right)\delta^\beta\Phi$$

Bossard, Cederwall, Kleinschmidt, Palmkvist, Samtleben 2017

$\eta_{k\,\alpha\beta}$: invariant bilinear form over $\hat{\mathfrak{e}}_8 \oplus \langle L_k\rangle$

- Closure of the generalised Lie derivative requires to impose a section constraint.

$$\boxed{Y^{MN}_{PQ}\,\partial_M\Phi\;\partial_N\Phi' = 0}$$

$\hat{E}_8 \rtimes \mathrm{Vir}^-$ covariant tensor

$\xrightarrow{\text{solve}}$ $\partial_M = 0$ $\longrightarrow$ ExFT reproduces: D=2 ungauged SUGRA

$$\partial_M = \left(\frac{\partial}{\partial y^i}\,,\,0\,,\,0\ldots\right) \longrightarrow$$

ExFT reproduces:

Highly non-trivial

D=11 SUGRA in a 2+9 KK split

or

D=10 IIB SUGRA in a 2+8 KK split

Generalised diffeomorphisms generate the internal diffeomorphisms and internal p-form gauge transformations

ExFT building blocks

Out of the scalars fields $\mathcal{V}(x,y)$, we build:

- **External currents:** $\frac{1}{2}(\mathcal{D}_\mu \mathcal{V}\mathcal{V}^{-1} + \text{h.c}) = \mathcal{P}_{\mu\,\alpha} \otimes T^\alpha$ $\longrightarrow$ $\mathcal{P}_{\mu\,\alpha}$ covariant under generalised diffeomorphisms

  Also shifted version $\mathcal{P}^{(1)}_{\mu\,\alpha}$

  Using the covariant derivative $\boxed{\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{|\mathcal{A}_\mu\rangle}}$

- **Internal currents:** $\frac{1}{2}(\partial_M \mathcal{V}\mathcal{V}^{-1} + \text{h.c}) = \mathcal{P}_{M\,\alpha} \otimes T^\alpha$ $\longrightarrow$ $\langle\mathcal{P}_\alpha|$ **not** covariant under generalised diffeomorphisms

Manifestly duality symmetric **dynamics encoded** in:

Pseudo Lagrangian $L_{\text{ExFT}} = L^{\text{top.}}_{\text{ExFT}} - \star V_{\text{ExFT}}$ **+** **Gauged twisted self-duality equation** $\star\mathcal{P} = \mathcal{P}^{(1)}$

Same trick as in gauged SUGRA

- Only depends on internal currents $\langle\mathcal{P}_\alpha|$.
- Fixed by generalised diffeomorphisms

Bossard, FC, Inverso, Kleinschmidt, Samtleben 2019/21

ExFT dynamics

Complete dynamics encoded into: $L_{\text{ExFT}} = L^{\text{top.}}_{\text{ExFT}} - \star V_{\text{ExFT}}$ **+** $\star\mathcal{P} = \mathcal{P}^{(1)}$

$$\rho^{-1} V_{\text{ExFT}} = \tfrac{1}{2}\,\eta_0^{\alpha\beta}\langle\mathcal{P}_\alpha|\mathcal{V}^{-1}\mathcal{V}^{-\dagger}|\mathcal{P}_\beta\rangle - \langle\mathcal{P}_\alpha|\mathcal{V}^{-1}T^\alpha T^\beta\mathcal{V}^{-\dagger}|\mathcal{P}_\beta\rangle + \langle\mathcal{P}^{(1)}_\alpha|\mathcal{V}^{-1}T^\alpha T^\beta\mathcal{V}^{-\dagger}|\mathcal{P}^{(1)}_\beta\rangle$$

$$+\langle\mathcal{P}_\alpha|\mathcal{V}^{-1}T^\alpha\mathcal{V}^{-\dagger}|\mathcal{P}_{L_0}\rangle + 2\sum_{q=1}^{\infty}\left(\langle\mathcal{P}_\alpha|\mathcal{V}^{-1}T^\alpha\mathcal{V}^{-\dagger}|\mathcal{P}_{L_q}\rangle + q(q-1)\langle\mathcal{P}_{L_q}|\mathcal{V}^{-1}\mathcal{V}^{-\dagger}|\mathcal{P}_{L_q}\rangle\right)$$

⚠ *All fields depend on the full set of coordinates* (x^μ, y^M).

<u>*Check*</u>: ***Reproduces D=11 or IIB supergravity*** dynamics upon choosing corresponding solutions to the section constraint:

$$\partial_M = \left(\frac{\partial}{\partial y^i}\,,\,0\,,\,0\ldots\right) \longrightarrow$$

D=11 SUGRA
in a 2+9 KK split

or

D=10 IIB SUGRA
in a 2+8 KK split

Highly non-trivial

Appropriate setup to study Kaluza-Klein truncations to D=2 gauged supergravities.

GSS truncations to D=2 gauged supergravities

Consider the simple truncation Ansatz:

$$\mathcal{V}(x,y) = V(x)\, U(y)$$
$$|\mathcal{A}_\mu(x,y)\rangle = U^{-1}(y)\, |A_\mu(x)\rangle$$
$$|\Lambda(x,y)\rangle = U^{-1}(y)\, |\lambda(x)\rangle$$
$$\vdots$$

ExFT fields and gauge parameters

D=2 gSUGRA fields and gauge parameters

with

'Twist matrix'

$U(y) \in \hat{E}_8 \rtimes \mathrm{Vir}^-$

respects the section constraint

Assume twist matrix $U(y)$ such that:

$$\mathcal{L}_{|\Lambda\rangle}\mathcal{V} \sim \Big(\langle\Theta_\alpha|\lambda\rangle\, \delta^\alpha V\Big)\, U$$
$$\vdots$$

Generalised diffeomorphisms in ExFT

Gauge transformation in gSUGRA

with **constant** embedding tensor $\langle\Theta_\alpha|$

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$$\vdots$$

Generalised diffeomorphisms in ExFT

Gauge transformation in gSUGRA

with **constant** embedding tensor $\langle\Theta_\alpha|$

Then

$$L_{\mathrm{ExFT}} = L^{\mathrm{top.}}_{\mathrm{ExFT}} - \star V_{\mathrm{ExFT}} \longrightarrow [U(y)]_{L_0}\, L_{\mathrm{gSUGRA}} = [U(y)]_{L_0}\Big(L^{\mathrm{top.}}_{\mathrm{gsugra}} - \star V_{\mathrm{gsugra}}\Big)$$

$$\star\mathcal{P} = \mathcal{P}^{(1)} \longrightarrow \star\, P = P^{(1)}$$

Consistent truncation!

D=2 maximal gauged supergravities

We find:

$$L^{\text{top.}}_{\text{gsugra}} = \rho \mathbf{D}\chi + \mathcal{O}\Big(V, \langle\Theta_\alpha|, |F\rangle\Big)$$

Recover previous D=2 result

$$V_{\text{gsugra}} = \frac{1}{2\rho^3}\langle\theta|M^{-1}|\theta\rangle + \frac{1}{2\rho}\eta_{-2\,\alpha\beta}\langle\theta|T^\alpha M^{-1}T^{\beta\dagger}|\theta\rangle$$

with

- Supergravity scalars packaged in $M(x) = V^\dagger V$
- $\eta_{k\,\alpha\beta}$: invariant bilinear form over $\hat{\mathfrak{e}}_8 \oplus \langle L_k\rangle$
- Constant embedding tensor $\langle\Theta_\alpha| = \eta_{-1\,\alpha\beta}\langle\theta|T^\beta$

→ *All fields only depend on D=2 coordinates*

→ *Gauge invariant D=2 pseudo-Lagrangian*

✓ *Full dynamics of all D=2 gauged maximal supergravities that result from consistent truncations D=11/10 SUGRA*

Example: sphere truncation

- We explicitly constructed the twist matrix $U(y)$ which leads to a constant embedding tensor that induces an $SO(9)$ gauging in D=2.

  Depends on 8 coordinates

→ Consistent truncation of D=10 IIA supergravity on S^8 to $SO(9)$ gauged supergravity

$$\boxed{SO(9)} \subset SL(9) \subset \widehat{SL}(9) \subset E_9$$

$$\cancel{SO(9)} \subset SL(9) \subset E_8 \subset E_9$$

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- *Rich structure of vacua to explore:*

Simplest D=2 solution: SO(9)-invariant half-supersymmetric domain wall

→ $\text{coset scalars} = \text{cst}$ $\quad$ $\text{gauge fields} = 0$ $\quad$ $ds_2^2 = r^7 dt^2 - dr^2$ $\quad$ $\rho = r^{9/2}$

Uplifts to solution of IIA supergravity: warped $AdS_2 \times S^8$ geometry with flux.

→ Near-horizon geometry of D0 branes: dual to *SU(N)* matrix quantum mechanics.

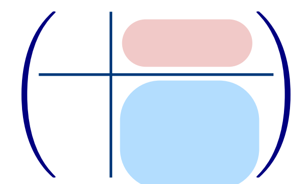
Holographic access to supermembrane/ BFSS model

Thank you for your attention.

Two paths to D=2

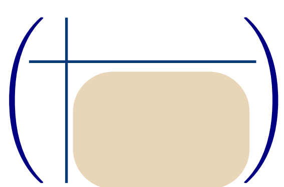
D=11: Elfbein

T^9 T^8



D=3: $R^+ \times SL(8)$ symmetry

Dualisation *of the 8 KK vectors into scalars* $\Longrightarrow SL(9)$ symmetry



D=2: $R^+ \times SL(9)$ symmetry $\Longleftrightarrow$ $R^+ \times SL(9)$ symmetry

Realised on scalars dual to each other

Two (on-shell) equivalent versions of the D=2 theory

Realising the two *SL(9)* simultaneously requires an infinite number of dual scalars

“ $SL(9) \times SL(9) = \widetilde{SL(9)}$ ” loop group